HEADS-UP

REACH FOR THE STARS

IGCSE

ADDITIONAL MATHEMATICS

TOPICAL PRACTICE QUESTIONS

PAPER 1

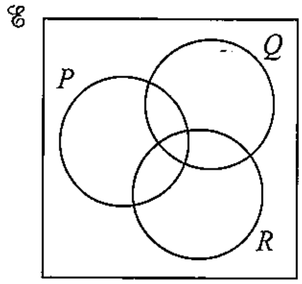
VOL. 2

2012-2015

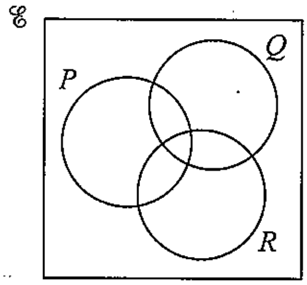
STUDENT COPY

2020 EDITION

1 **(a)** On the Venn diagrams below, shade the region corresponding to the set given below each Venn diagram.



$P \cup (Q \cap R)$



$P \cap (Q \cup R)$

[2]

(b) It is given that sets $\mathscr{E}$, B, S and F are such that

$\mathscr{E}$ = {students in a school},

B = {students who are boys},

S = {students in the swimming team},

F = {students in the football team}.

Express each of the following statements in set notation.

(i) All students in the football team are boys. [1]

(ii) There are no students who are in both the swimming team and the football team. [1]

2 The sets A and B are such that

$$A = \left\{x : \cos x = \frac{1}{2}, 0° \leqslant x \leqslant 620°\right\},$$

$$B = \{x : \tan x = \sqrt{3}, 0° \leqslant x \leqslant 620°\}.$$

(i) Find n(A). [1]

(ii) Find n(B). [1]

(iii) Find the elements of $A \cup B$. [1]

(iv) Find the elements of $A \cap B$. [1]

Paper 1 - Oct Nov 2013 Code 11,12

3 The universal set $\mathscr{E}$ is the set of real numbers. Sets A, B and C are such that

$$A = \{x : x^2 + 5x + 6 = 0\},$$

$$B = \{x : (x-3)(x+2)(x+1) = 0\},$$

$$C = \{x : x^2 + x + 3 = 0\}.$$

(i) State the value of each of n(A), n(B) and n(C). [3]

(ii) List the elements in the set $A \cup B$. [1]

(iii) List the elements in the set $A \cap B$. [1]

(iv) Describe the set C'. [1]

Paper 1 - Oct Nov 2014 Code 13

4 It is given that $\mathscr{E} = \{x : 1 \leqslant x \leqslant 12, \text{where } x \text{ is an integer}\}$ and that sets A, B, C and D are such that

$A = \{\text{multiples of } 3\}$,
$B = \{\text{prime numbers}\}$,
$C = \{\text{odd integers}\}$,
$D = \{\text{even integers}\}$.

Write down the following sets in terms of their elements.

(i) $A \cap B$ [1]

(ii) $A \cup C$ [1]

(iii) $A' \cap C$ [1]

(iv) $(D \cup B)'$ [1]

(v) Write down a set E such that $E \subset D$. [1]

Paper 1 - Oct Nov 2015 Code 11

1 Find the set of values of k for which the line $y = 2x + k$ cuts the curve $y = x^2 + kx + 5$ at two distinct points. [6]

May June 2012 Code 11,13

2 Solve the simultaneous equations $5x + 3y = 2$ and $\frac{2}{x} - \frac{3}{y} = 1$. [5]

May June 2012 Code 12

3 The line $x - 2y = 6$ intersects the curve $x^2 + xy + 10y + 4y^2 = 156$ at the points A and B.
Find the length of AB. [7]

Oct Nov 2012 Code 12

4 Find the values of k for which the line $y = k - 6x$ is a tangent to the curve $y = x(2x + k)$. [4]

Oct Nov 2012 Code 11

5 The line $3x + 4y = 15$ cuts the curve $2xy = 9$ at the points A and B. Find the length of the line AB. [6]

May June 2013 Code 12

6 Find the set of values of k for which the line $y = k(4x - 3)$ does not intersect the curve $y = 4x^2 + 8x - 8$. [5]

May June 2014 Code 11

7 The line $4y = x + 8$ cuts the curve $xy = 4 + 2x$ at the points A and B. Find the exact length of AB. [5]

Oct Nov 2014 Code 13

8 The curve $y = xy + x^2 - 4$ intersects the line $y = 3x - 1$ at the points A and B. Find the equation of the perpendicular bisector of the line AB. [8]

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9 Find the range of values of k for which the equation $kx^2 + k = 8x - 2xk$ has 2 real distinct roots. [4]

Paper 1 - Oct Nov 2015 Code 11

10 The line $x - y + 2 = 0$ intersects the curve $2x^2 - y^2 + 2x + 1 = 0$ at the points A and B. The perpendicular bisector of the line AB intersects the curve at the points C and D. Find the length of the line CD in the form $a\sqrt{5}$, where a is an integer. [10]

Paper 1 - Oct Nov 2015 Code 13

1 **(a)** Find the value of x for which $2\lg x - \lg(5x + 60) = 1$. [5]

(b) Solve $\log_5 y = 4\log_y 5$. [4]

May June 2012 Code 11,13

2 **You must not use a calculator in this question.**

(i) Express $\frac{8}{\sqrt{3}+1}$ in the form $a(\sqrt{3}-1)$, where a is an integer. [2]

An equilateral triangle has sides of length $\frac{8}{\sqrt{3}+1}$.

(ii) Show that the height of the triangle is $6-2\sqrt{3}$. [2]

(iii) Hence, or otherwise, find the area of the triangle in the form $p\sqrt{3}-q$, where p and q are integers. [2]

May June 2012 Code 12

3 **Do not use a calculator in any part of this question.**

(a) **(i)** Show that $3\sqrt{5} - 2\sqrt{2}$ is a square root of $53 - 12\sqrt{10}$. [1]

(ii) State the other square root of $53 - 12\sqrt{10}$. [1]

(b) Express $\dfrac{6\sqrt{3} + 7\sqrt{2}}{4\sqrt{3} + 5\sqrt{2}}$ in the form $a + b\sqrt{6}$, where a and b are integers to be found. [4]

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4 Given that $p = \log_q 32$, express, in terms of p,

(i) $\log_q 4$, [2]

(ii) $\log_q 16q$. [2]

Oct Nov 2012 Code 11

5 Given that $\log_a pq = 9$ and $\log_a p^2q = 15$, find the value of

(i) $\log_a p$ and of $\log_a q$, [4]

(ii) $\log_p a + \log_q a$. [2]

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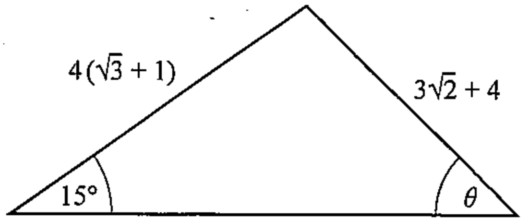
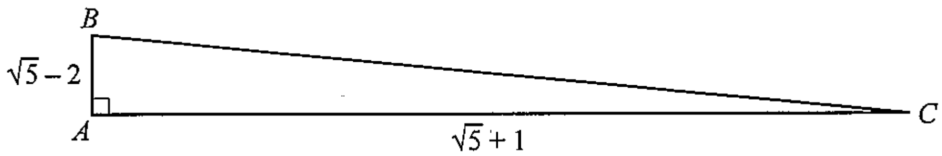
6



Using $\sin 15° = \frac{\sqrt{2}}{4}(\sqrt{3}-1)$ and without using a calculator, find the value of $\sin\theta$ in the form $a + b\sqrt{2}$, where a and b are integers. [5]

Oct Nov 2012 Code 12

7 **Calculators must not be used in this question.**



The diagram shows a triangle ABC in which angle $A = 90°$. Sides AB and AC are $\sqrt{5} - 2$ and $\sqrt{5} + 1$ respectively. Find

(i) $\tan B$ in the form $a + b\sqrt{5}$, where a and b are integers, [3]

(ii) $\sec^2 B$ in the form $c + d\sqrt{5}$, where c and d are integers. [4]

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8 **(i)** Given that $\log_4 x = \frac{1}{2}$, find the value of x. [1]

(ii) Solve $2\log_4 y - \log_4(5y - 12) = \frac{1}{2}$. [4]

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9 Solve $2\lg y - \lg(5y + 60) = 1$. [5]

Oct Nov 2013 Code 13

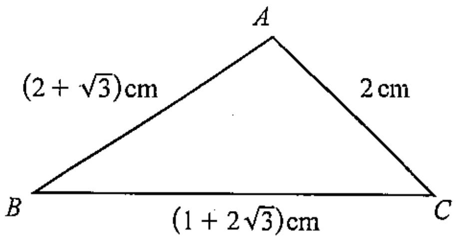
10 **(a)** Solve the following simultaneous equations.

$$\frac{5^x}{25^{3y-2}} = 1$$

$$\frac{3^x}{27^{y-1}} = 81$$

[5]

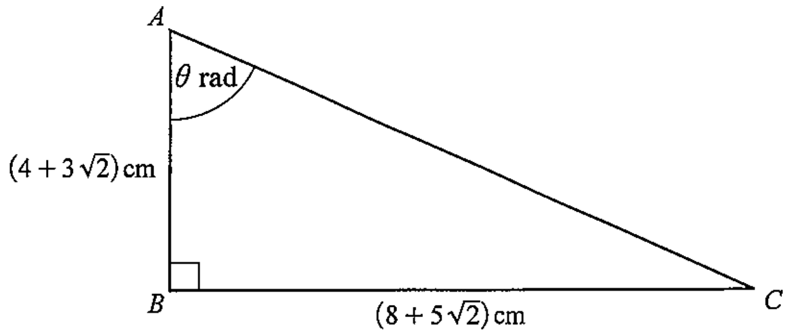
(b) The diagram shows a triangle ABC such that $AB = (2+\sqrt{3})$ cm, $BC = (1+2\sqrt{3})$ cm and $AC = 2$ cm.



Without using a calculator, find the value of $\cos A$ in the form $a+b\sqrt{3}$, where a and b are constants to be found. [4]

Oct Nov 2014 Code 13

11 **Do not use a calculator in this question.**



The diagram shows the triangle ABC where angle B is a right angle, $AB = (4 + 3\sqrt{2})$ cm, $BC = (8 + 5\sqrt{2})$ cm and angle $BAC = \theta$ radians. Showing all your working, find

(i) $\tan\theta$ in the form $a + b\sqrt{2}$, where a and b are integers, [2]

(ii) $\sec^2\theta$ in the form $c + d\sqrt{2}$, where c and d are integers. [3]

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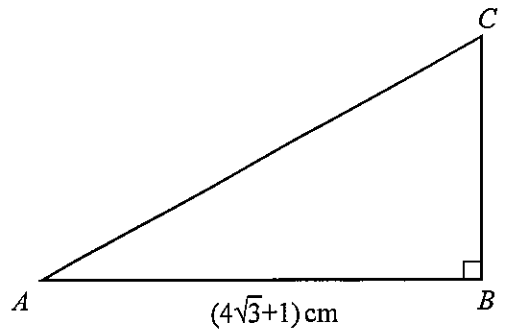
12 Solve the equation $1 + 2\log_5 x = \log_5(18x - 9)$. [5]

Oct Nov 2014 Code 13

13 **You are not allowed to use a calculator in this question.**



The diagram shows triangle ABC with side $AB = (4\sqrt{3} + 1)$ cm. Angle B is a right angle. It is given that the area of this triangle is $\frac{47}{2}$ cm^2.

(i) Find the length of the side BC in the form $(a\sqrt{3} + b)$ cm, where a and b are integers. [3]

(ii) Hence find the length of the side AC in the form $p\sqrt{2}$ cm, where p is an integer. [2]

Paper 1 - Oct Nov 2015 Code 13

14 **(a)** Given that $2^{2x-1} \times 4^{x+y} = 128$ and $\frac{9^{2y-x}}{27^{y-4}} = 1$, find the value of each of the integers x and y. [4]

Paper 1 - Oct Nov 2015 Code 13

1 The expression $2x^3 + ax^2 + bx - 30$ is divisible by $x + 2$ and leaves a remainder of -35 when divided by $2x - 1$. Find the values of the constants a and b. [5]

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2 A function f is such that $f(x) = 4x^3 + 4x^2 + ax + b$. It is given that $2x - 1$ is a factor of both $f(x)$ and $f'(x)$.

(i) Show that $b = 2$ and find the value of a. [5]

Using the values of a and b from part **(i)**,

(ii) find the remainder when $f(x)$ is divided by $x + 3$, [2]

(iii) express $f(x)$ in the form $f(x) = (2x - 1)(px^2 + qx + r)$, where p, q and r are integers to be found, [2]

(iv) find the values of x for which $f(x) = 0$. [2]

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3 **(i)** Given that $3x^3 + 5x^2 + px + 8 \equiv (x-2)(ax^2 + bx + c)$, find the value of each of the integers a, b, c and p. [5]

(ii) Using the values found in part **(i)**, factorise completely $3x^3 + 5x^2 + px + 8$. [2]

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4 It is given that $f(x) = 6x^3 - 5x^2 + ax + b$ has a factor of $x + 2$ and leaves a remainder of 27 when divided by $x - 1$.

(i) Show that $b = 40$ and find the value of a. [4]

(ii) Show that $f(x) = (x + 2)(px^2 + qx + r)$, where p, q and r are integers to be found. [2]

(iii) Hence solve $f(x) = 0$. [2]

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5 The function $f(x) = ax^3 + 4x^2 + bx - 2$, where a and b are constants, is such that $2x - 1$ is a factor. Given that the remainder when $f(x)$ is divided by $x - 2$ is twice the remainder when $f(x)$ is divided by $x + 1$, find the value of a and of b. [6]

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6 (i) Find, in terms of p, the remainder when $x^3 + px^2 + p^2x + 21$ is divided by $x + 3$. [2]

(ii) Hence find the set of values of p for which this remainder is negative. [3]

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7 The polynomial $f(x) = ax^3 - 15x^2 + bx - 2$ has a factor of $2x - 1$ and a remainder of 5 when divided by $x - 1$.

(i) Show that $b = 8$ and find the value of a. [4]

(ii) Using the values of a and b from part **(i)**, express $f(x)$ in the form $(2x - 1)\,g(x)$, where $g(x)$ is a quadratic factor to be found. [2]

(iii) Show that the equation $f(x) = 0$ has only one real root. [2]

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1 **(i)** Find the inverse of the matrix $\begin{pmatrix} 2 & -1 \\ -1 & 1.5 \end{pmatrix}$. [2]

(ii) Hence find the matrix **A** such that $\begin{pmatrix} 2 & -1 \\ -1 & 1.5 \end{pmatrix}\mathbf{A} = \begin{pmatrix} 1 & 6 \\ -0.5 & 4 \end{pmatrix}$. [3]

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2 **(i)** Given that $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}$, find $\mathbf{A}^{-1}$. [2]

(ii) Using your answer from part **(i)**, or otherwise, find the values of a, b, c and d such that

$$\mathbf{A}\begin{pmatrix} a & b \\ c & -1 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ 17 & d \end{pmatrix}.$$ [5]

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3 (a) Given that the matrix $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 3 & -5 \end{pmatrix}$, find

(i) $\mathbf{A}^2$, [2]

(ii) $3\mathbf{A} + 4\mathbf{I}$, where $\mathbf{I}$ is the identity matrix. [2]

(b) (i) Find the inverse matrix of $\begin{pmatrix} 6 & 1 \\ -9 & 3 \end{pmatrix}$. [2]

(ii) Hence solve the equations

$$6x + y = 5,$$
$$-9x + 3y = \frac{3}{2}.$$ [3]

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4 (a) It is given that the matrix $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$.

(i) Find $\mathbf{A} + 2\mathbf{I}$. [1]

(ii) Find $\mathbf{A}^2$. [2]

(iii) Using your answer to part (ii) find the matrix $\mathbf{B}$ such that $\mathbf{A}^2\mathbf{B} = \mathbf{I}$. [2]

(b) Given that the matrix $\mathbf{C} = \begin{pmatrix} x & -1 \\ x^2 - x + 1 & x - 1 \end{pmatrix}$, show that $\det \mathbf{C} \neq 0$. [4]

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5 It is given that $\mathbf{A} = \begin{pmatrix} 2t & 2 \\ t^2 - t + 1 & t \end{pmatrix}$.

(i) Find the value of t for which $\det \mathbf{A} = 1$. [3]

(ii) In the case when $t = 3$, find $\mathbf{A}^{-1}$ and hence solve

$$3x + y = 5,$$
$$7x + 3y = 11.$$

[5]

Oct Nov 2013 Code 13

6 Matrices **A** and **B** are such that $\mathbf{A} = \begin{pmatrix} -1 & 4 \\ 7 & 6 \\ 4 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$.

(i) Find **AB**. [2]

(ii) Find $\mathbf{B}^{-1}$. [2]

(iii) Using your answer to part **(ii)**, solve the simultaneous equations

$$4x + 2y = -3,$$
$$6x + 10y = -22.$$

[3]

May June 2014 Code 11

7 **(a)** Matrices **X**, **Y** and **Z** are such that $\mathbf{X} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, $\mathbf{Y} = \begin{pmatrix} 1 & 3 \\ 4 & 5 \\ 6 & 7 \end{pmatrix}$ and $\mathbf{Z} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$. Write down all the matrix products which are possible using any two of these matrices. Do not evaluate these products. [2]

(b) Matrices **A** and **B** are such that $\mathbf{A} = \begin{pmatrix} 5 & -2 \\ -4 & 1 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} 3 & 9 \\ -6 & -3 \end{pmatrix}$. Find the matrix **B**. [5]

May June 2014 Code 12

8 Matrices **A** and **B** are such that $\mathbf{A} = \begin{pmatrix} 3a & 2b \\ -a & b \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -a & b \\ 2a & 2b \end{pmatrix}$, where a and b are non-zero constants.

(i) Find $\mathbf{A}^{-1}$. [2]

(ii) Using your answer to part **(i)**, find the matrix **X** such that $\mathbf{XA} = \mathbf{B}$. [4]

Oct Nov 2014 Code 11,12

9 (a) A drinks machine sells coffee, tea and cola. Coffee costs \$0.50, tea costs \$0.40 and cola costs \$0.45. The table below shows the numbers of drinks sold over a 4-day period.

	Coffee	Tea	Cola
Tuesday	12	2	1
Wednesday	9	3	0
Thursday	8	5	1
Friday	11	2	0

(i) Write down 2 matrices whose product will give the amount of money the drinks machine took each day and evaluate this product. [4]

(ii) Hence write down the total amount of money taken by the machine for this 4-day period. [1]

(b) Matrices **X** and **Y** are such that $\mathbf{X} = \begin{pmatrix} 2 & 4 \\ -5 & 1 \end{pmatrix}$ and $\mathbf{XY} = \mathbf{I}$, where **I** is the identity matrix. Find the matrix **Y**. [3]

Oct Nov 2014 Code 13

10 **(a)** Given that the matrix $\mathbf{X} = \begin{pmatrix} 2 & -4 \\ k & 0 \end{pmatrix}$, find $\mathbf{X}^2$ in terms of the constant k. [2]

(b) Given that the matrix $\mathbf{A} = \begin{pmatrix} a & 1 \\ b & 5 \end{pmatrix}$ and the matrix $\mathbf{A}^{-1} = \begin{pmatrix} \frac{5}{6} & -\frac{1}{6} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$, find the value of each of the integers a and b. [3]

May June 2015 Code 11,13

11 Find the inverse of the matrix $\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$ and hence solve the simultaneous equations

$$4x + 2y - 8 = 0,$$
$$5x + 3y - 9 = 0.$$

[5]

May June 2015 Code 12

12 **(a)** Matrices **A** and **B** are such that $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 8 & 1 \\ 6 & 0 & 2 \end{pmatrix}$. Find **AB**. [2]

Paper 1 - Oct Nov 2015 Code 13

(b) Given that matrix $\mathbf{X} = \begin{pmatrix} 4 & 6 \\ 2 & -8 \end{pmatrix}$, find the integer value of m and of n such that $\mathbf{X}^2 = m\mathbf{X} + n\mathbf{I}$, where **I** is the identity matrix. [5]

(c) Given that matrix $\mathbf{Y} = \begin{pmatrix} a & 2 \\ 3 & a \end{pmatrix}$, find the values of a for which det $\mathbf{Y} = 0$. [2]

1 The point P lies on the line joining $A(-1, -5)$ and $B(11, 13)$ such that $AP = \frac{1}{3}AB$.

(i) Find the equation of the line perpendicular to AB and passing through P. [5]

The line perpendicular to AB passing through P and the line parallel to the x-axis passing through B intersect at the point Q.

(ii) Find the coordinates of the point Q. [2]

(iii) Find the area of the triangle PBQ. [2]

May June 2012 Code 12

2 The points $A(-3, 6)$, $B(5, 2)$ and C lie on a straight line such that B is the mid-point of AC.

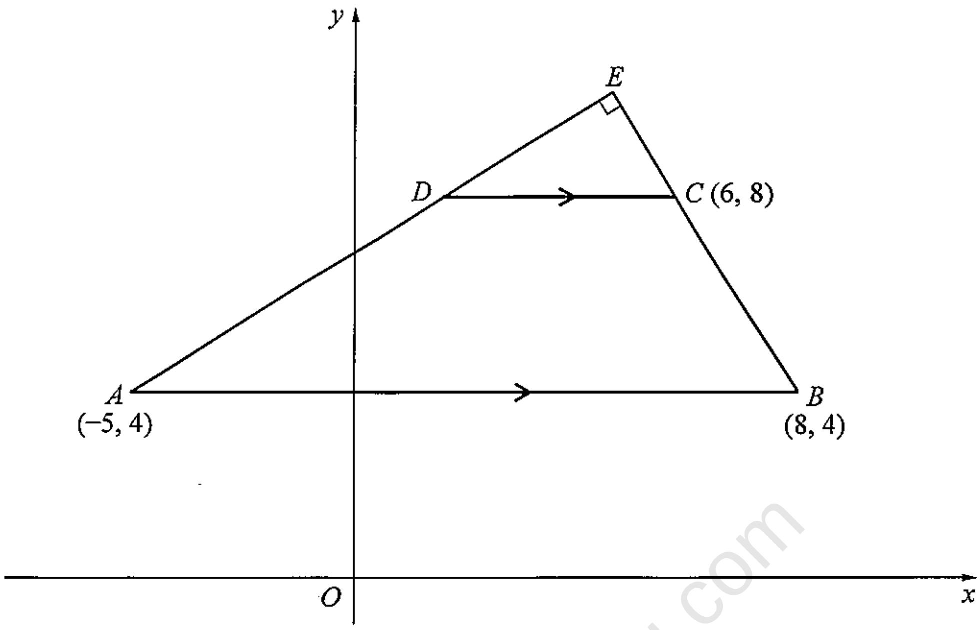
(i) Find the coordinates of C. [2]

The point D lies on the y-axis and the line CD is perpendicular to AC.

(ii) Find the area of the triangle ACD. [5]

Oct Nov 2012 Code 11

3 **Solutions to this question by accurate drawing will not be accepted.**



The vertices of the trapezium $ABCD$ are the points $A(-5, 4)$, $B(8, 4)$, $C(6, 8)$ and D. The line AB is parallel to the line DC. The lines AD and BC are extended to meet at E and angle $AEB = 90°$.

(i) Find the coordinates of D and of E. [6]

(ii) Find the area of the trapezium $ABCD$. [2]

Oct Nov 2012 Code 12

4 **Solutions to this question by accurate drawing will not be accepted.**

The points $A(-3, 2)$ and $B(1, 4)$ are vertices of an isosceles triangle ABC, where angle $B = 90°$.

(i) Find the length of the line AB. [1]

(ii) Find the equation of the line BC. [3]

(iii) Find the coordinates of each of the two possible positions of C. [6]

Oct Nov 2013 Code 11,12

5 The point P lies on the line joining $A(-2, 3)$ and $B(10, 19)$ such that $AP:PB = 1:3$.

(i) Show that the x-coordinate of P is 1 and find the y-coordinate of P. [2]

(ii) Find the equation of the line through P which is perpendicular to AB. [3]

The line through P which is perpendicular to AB meets the y-axis at the point Q.

(iii) Find the area of the triangle AQB. [3]

Oct Nov 2014 Code 11,12

6 The line $2x - y + 1 = 0$ meets the curve $x^2 + 3y = 19$ at the points A and B. The perpendicular bisector of the line AB meets the x-axis at the point C. Find the area of the triangle ABC. [9]

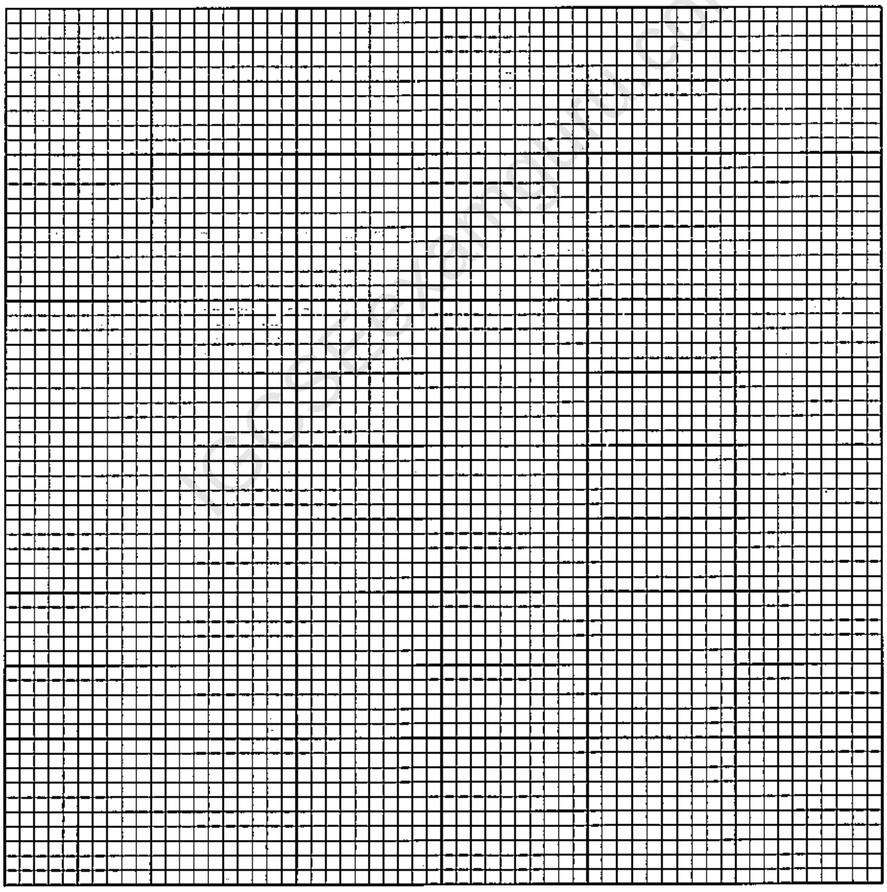
Paper 1 - Oct Nov 2015 Code 11

1 The table shows values of the variables x and y.

x	10°	30°	45°	60°	80°
y	11.2	16	19.5	22.4	24.7

(i) Using the graph paper below, plot a suitable straight line graph to show that, for $10° \leqslant x \leqslant 80°$,

$\sqrt{y} = A\sin x + B$, where A and B are positive constants. [4]



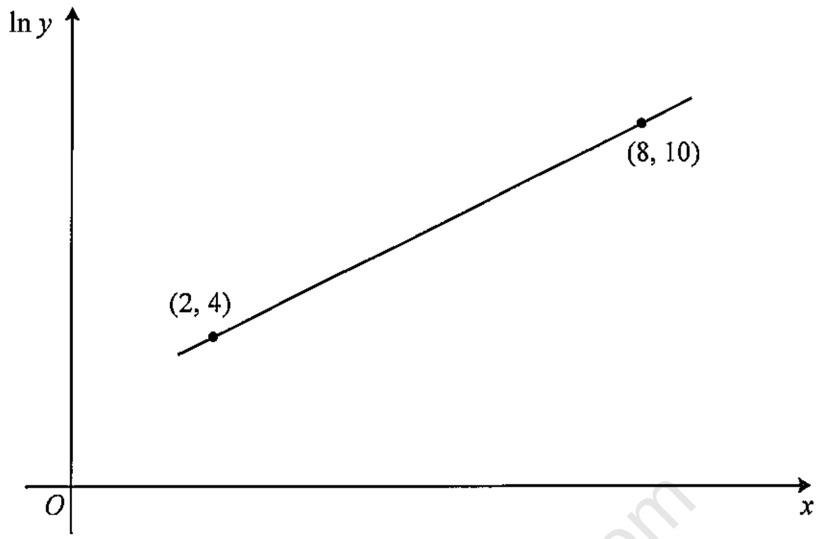
(ii) Use your graph to find the value of A and of B. [3]

(iii) Estimate the value of y when $x = 50$. [2]

(iv) Estimate the value of x when $y = 12$. [2]

Oct Nov 2012 Code 11

2 Variables x and y are such that $y = Ab^x$, where A and b are constants. The diagram shows the graph of $\ln y$ against x, passing through the points (2, 4) and (8, 10).



Find the value of A and of b. [5]

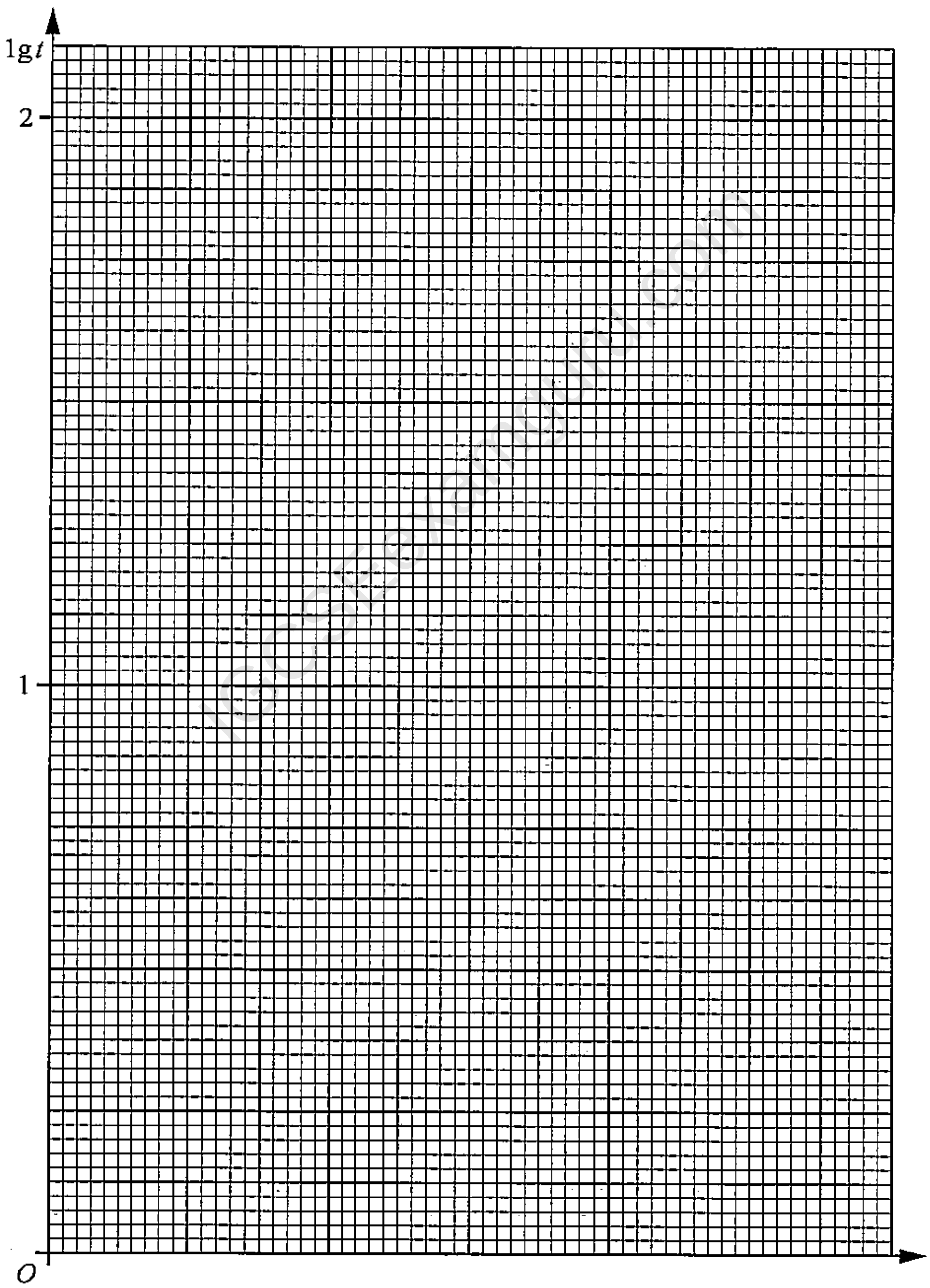
May June 2013 Code 11,13

3 The variables s and t are related by the equation $t = ks^n$, where k and n are constants. The table below shows values of variables s and t.

s	2	4	6	8
t	25.00	6.25	2.78	1.56

(i) A straight line graph is to be drawn for this information with $\lg t$ plotted on the vertical axis. State the variable which must be plotted on the horizontal axis. [1]

(ii) Draw this straight line graph on the grid below. [3]



(iii) Use your graph to find the value of k and of n. [4]

(iv) Estimate the value of s when $t = 4$. [2]

Oct Nov 2013 Code 13

4 The table shows values of variables V and p.

V	10	50	100	200
p	95.0	8.5	3.0	1.1

(i) By plotting a suitable straight line graph, show that V and p are related by the equation $p = kV^n$, where k and n are constants. [4]

Use your graph to find

(ii) the value of n, [2]

(iii) the value of p when $V = 35$. [2]

May June 2014 Code 11

5 The table shows experimental values of x and y.

x	1.50	1.75	2.00	2.25
y	3.9	8.3	19.5	51.7

(i) Complete the following table.

x^2				
$\lg y$				

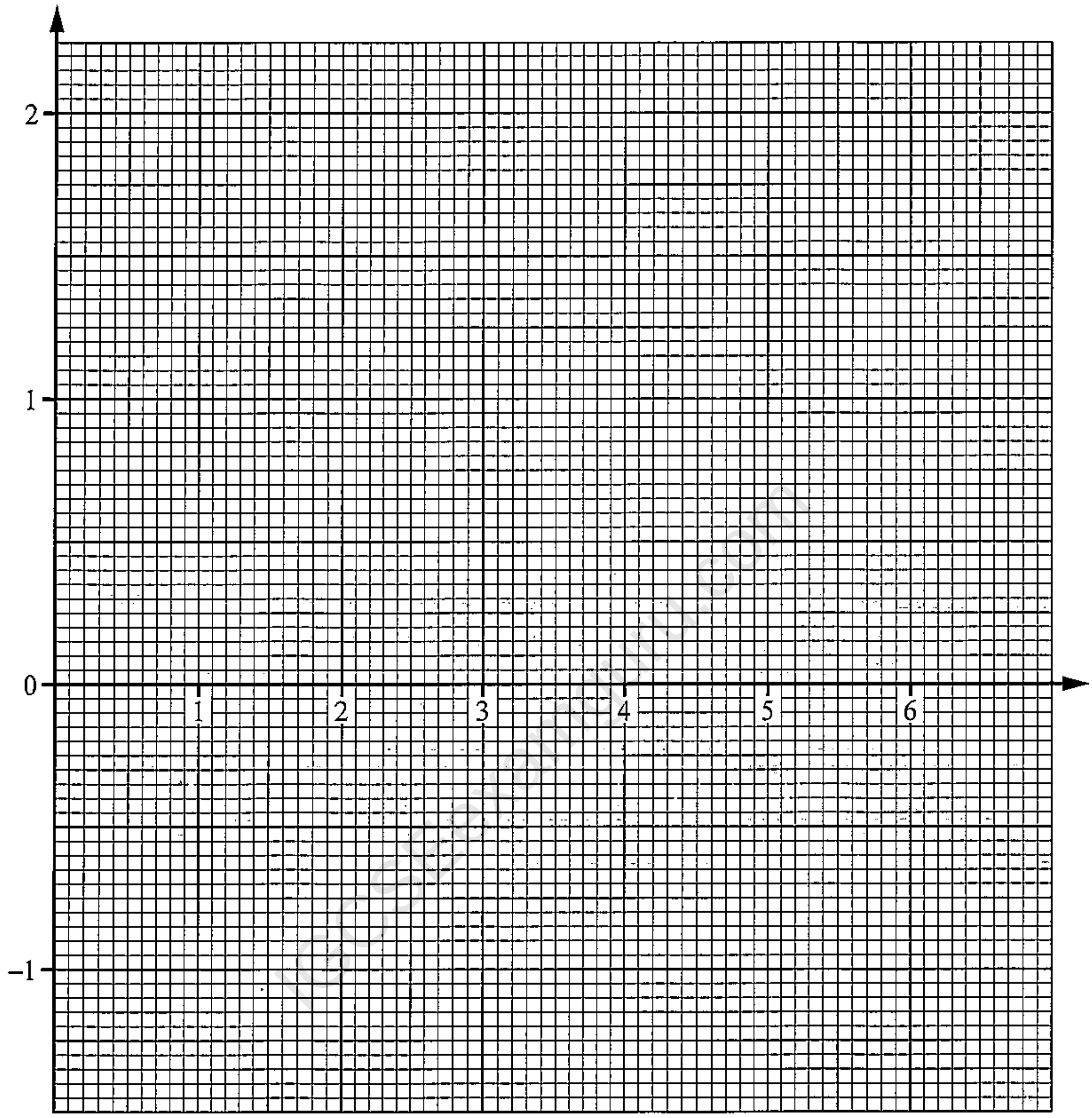
[1]

(ii) By plotting a suitable straight line graph on the grid on page 13, show that x and y are related by the equation $y = Ab^{x^2}$, where A and b are constants. [2]

(iii) Use your graph to find the value of A and of b. [4]

(iv) Estimate the value of y when $x = 1.25$. [2]

May June 2014 Code 13


2
1
0
−1
1
2
3
4
5
6

6 The table shows experimental values of variables x and y.

x	2	2.5	3	3.5	4
y	18.8	29.6	46.9	74.1	117.2

(i) By plotting a suitable straight line graph on the grid below, show that x and y are related by the equation $y = ab^x$, where a and b are constants. [4]

(ii) Use your graph to find the value of a and of b. [4]

Oct Nov 2014 Code **11, 12**

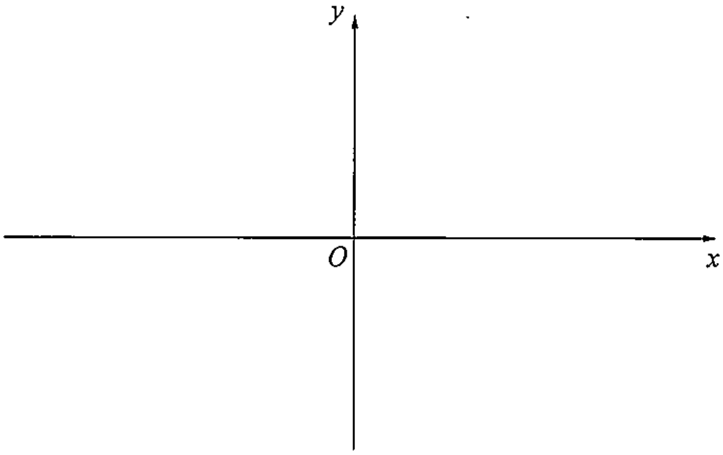
Two variables, x and y, are such that $y = Ax^b$, where A and b are constants. When $\ln y$ is plotted against $\ln x$, a straight line graph is obtained which passes through the points (1.4, 5.8) and (2.2, 6.0).

(i) Find the value of A and of b. [4]

(ii) Calculate the value of y when $x = 5$. [2]

Paper 1 - Oct Nov 2015 Code 11

1 (i) Sketch the graph of $y = |2x - 5|$, showing the coordinates of the points where the graph meets the coordinate axes. [2]



(ii) Solve $|2x - 5| = 3$. [2]

May June 2012 Code 11,13

2 (i) Sketch the graph of $y = |x^2 - x - 6|$, showing the coordinates of the points where the curve meets the coordinate axes. [3]

(ii) Solve $|x^2 - x - 6| = 6$. [3]

May June 2012 Code 12

3 (a) It is given that $f(x) = \frac{1}{2+x}$ for $x \neq -2, x \in \mathbb{R}$.

(i) Find $f''(x)$. [2]

(ii) Find $f^{-1}(x)$. [2]

(iii) Solve $f^2(x) = -1$. [3]

(b) The functions g, h and k are defined, for $x \in \mathbb{R}$, by

$$g(x) = \frac{1}{x+5}, \quad x \neq -5,$$
$$h(x) = x^2 - 1,$$
$$k(x) = 2x + 1.$$

Express the following in terms of g, h and/or k.

(i) $\frac{1}{(x^2-1)+5}$ [1]

(ii) $\frac{2}{x+5} + 1$ [1]

May June 2012 Code 12

4 (i) Sketch the graph of $y = |3 + 5x|$, showing the coordinates of the points where your graph meets the coordinate axes. [2]

(ii) Solve the equation $|3 + 5x| = 2$. [2]

Oct Nov 2012 Code 11

5 A function g is such that $g(x) = \frac{1}{2x-1}$ for $1 \leqslant x \leqslant 3$.

(i) Find the range of g. [1]

(ii) Find $g^{-1}(x)$. [2]

(iii) Write down the domain of $g^{-1}(x)$. [1]

(iv) Solve $g^2(x) = 3$. [3]

Oct Nov 2012 Code 11

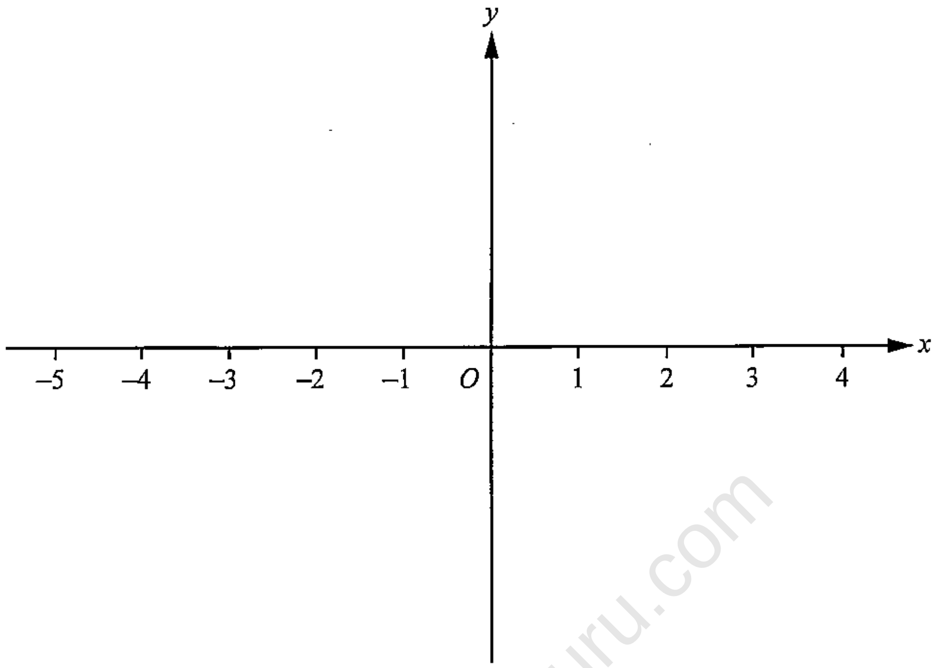
6 Find the set of values of k for which the curve $y = 2x^2 + kx + 2k - 6$ lies above the x-axis for all values of x. [4]

May June 2013 Code 12

7 Find the set of values of k for which the curve $y = (k+1)x^2 - 3x + (k+1)$ lies below the x-axis. [4]

Oct Nov 2013 Code 11,12

8 **(i)** On the grid below, sketch the graph of $y = |(x-2)(x+3)|$ for $-5 \leqslant x \leqslant 4$, and state the coordinates of the points where the curve meets the coordinate axes. [4]



(ii) Find the coordinates of the stationary point on the curve $y = |(x-2)(x+3)|$. [2]

(iii) Given that k is a positive constant, state the set of values of k for which $|(x-2)(x+3)| = k$ has 2 solutions only. [1]

Oct Nov 2013 Code 11,12

9 (a) A function f is such that $f(x) = 3x^2 - 1$ for $-10 \leqslant x \leqslant 8$.

(i) Find the range of f. [3]

(ii) Write down a suitable domain for f for which f^{-1} exists. [1]

(b) Functions g and h are defined by

$$g(x) = 4e^x - 2 \text{ for } x \in \mathbb{R},$$
$$h(x) = \ln 5x \text{ for } x > 0.$$

(i) Find $g^{-1}(x)$. [2]

(ii) Solve $gh(x) = 18$. [3]

Oct Nov 2013 Code 11,12

10 For $x \in \mathbb{R}$, the functions f and g are defined by

$$f(x) = 2x^3,$$

$$g(x) = 4x - 5x^2.$$

(i) Express $f^2\left(\frac{1}{2}\right)$ as a power of 2. [2]

(ii) Find the values of x for which f and g are increasing at the same rate with respect to x. [4]

Oct Nov 2013 Code 13

11 (i) Sketch the graph of $y = |(2x+1)(x-2)|$ for $-2 \leqslant x \leqslant 3$, showing the coordinates of the points where the curve meets the x- and y-axes. [3]

(ii) Find the non-zero values of k for which the equation $|(2x+1)(x-2)| = k$ has two solutions only. [2]

May June 2014 Code 12

12 **(i)** Show that $y = 3x^2 - 6x + 5$ can be written in the form $y = a(x - b)^2 + c$, where a, b and c are constants to be found. [3]

(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve $y = 3x^2 - 6x + 5$. [1]

May June 2014 Code 13

13 It is given that $f(x) = 3e^{2x}$ for $x \geqslant 0$,
$g(x) = (x+2)^2 + 5$ for $x \geqslant 0$.

(i) Write down the range of f and of g. [2]

(ii) Find g^{-1}, stating its domain. [3]

(iii) Find the exact solution of $gf(x) = 41$. [4]

(iv) Evaluate $f'(\ln 4)$. [2]

May June 2015 Code 11,13

14 Given that the graph of $y = (2k + 5)x^2 + kx + 1$ does not meet the x-axis, find the possible values of k. [4]

May June 2015 Code 12

15 **(a)** A function f is such that $f(x) = x^2 + 6x + 4$ for $x \geqslant 0$.

(i) Show that $x^2 + 6x + 4$ can be written in the form $(x + a)^2 + b$, where a and b are integers. [2]

(ii) Write down the range of f. [1]

(iii) Find f^{-1} and state its domain. [3]

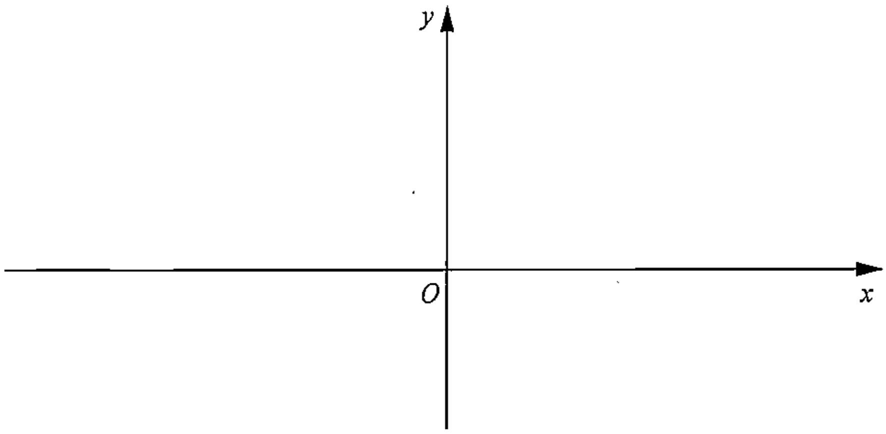
(b) Functions g and h are such that, for $x \in \mathbb{R}$,

$$g(x) = e^x \quad \text{and} \quad h(x) = 5x + 2.$$

Solve $h^2g(x) = 37$. [4]

Paper 1 - Oct Nov 2015 Code 11

16 **(i)** On the axes below, sketch the graph of $y = |x^2 - 4x - 12|$ showing the coordinates of the points where the graph meets the axes. [3]



(ii) Find the coordinates of the stationary point on the curve $y = |x^2 - 4x - 12|$. [2]

(iii) Find the values of k such that the equation $|x^2 - 4x - 12| = k$ has only 2 solutions. [2]

Paper 1 - Oct Nov 2015 Code 13

1 **(i)** Given that $15\cos^2\theta + 2\sin^2\theta = 7$, show that $\tan^2\theta = \frac{8}{5}$. [4]

(ii) Solve $15\cos^2\theta + 2\sin^2\theta = 7$ for $0 \leqslant \theta \leqslant \pi$ radians. [3]

May June 2012 Code 11,13

2 Show that $\cot A + \dfrac{\sin A}{1 + \cos A} = \operatorname{cosec} A$. [4]

May June 2012 Code 12

3 (a) Solve $\operatorname{cosec}\left(2x - \frac{\pi}{3}\right) = \sqrt{2}$ for $0 < x < \pi$ radians. [4]

(b) (i) Given that $5(\cos y + \sin y)(2\cos y - \sin y) = 7$, show that $12\tan^2 y - 5\tan y - 3 = 0$. [4]

(ii) Hence solve $5(\cos y + \sin y)(2\cos y - \sin y) = 7$ for $0° < x < 180°$. [3]

Oct Nov 2012 Code 11

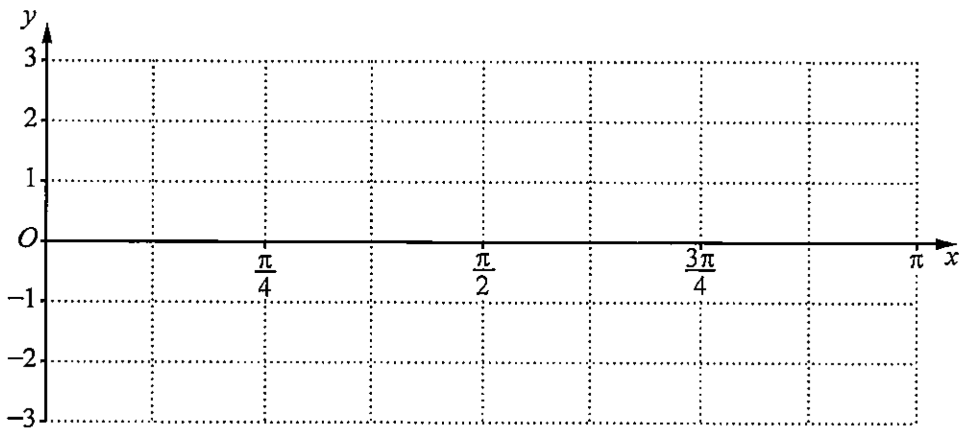
4 **(i)** Show that $\cot\theta + \dfrac{\sin\theta}{1+\cos\theta} = \operatorname{cosec}\theta$. [5]

(ii) Explain why the equation $\cot\theta + \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1}{2}$ has no solution. [1]

Oct Nov 2012 Code 12

5 **(a)** **(i)** Using the axes below, sketch for $0 \leqslant x \leqslant \pi$, the graphs of

$$y = \sin 2x \quad \text{and} \quad y = 1 + \cos 2x.$$ [4]



(ii) Write down the solutions of the equation $\sin 2x - \cos 2x = 1$, for $0 \leqslant x \leqslant \pi$. [2]

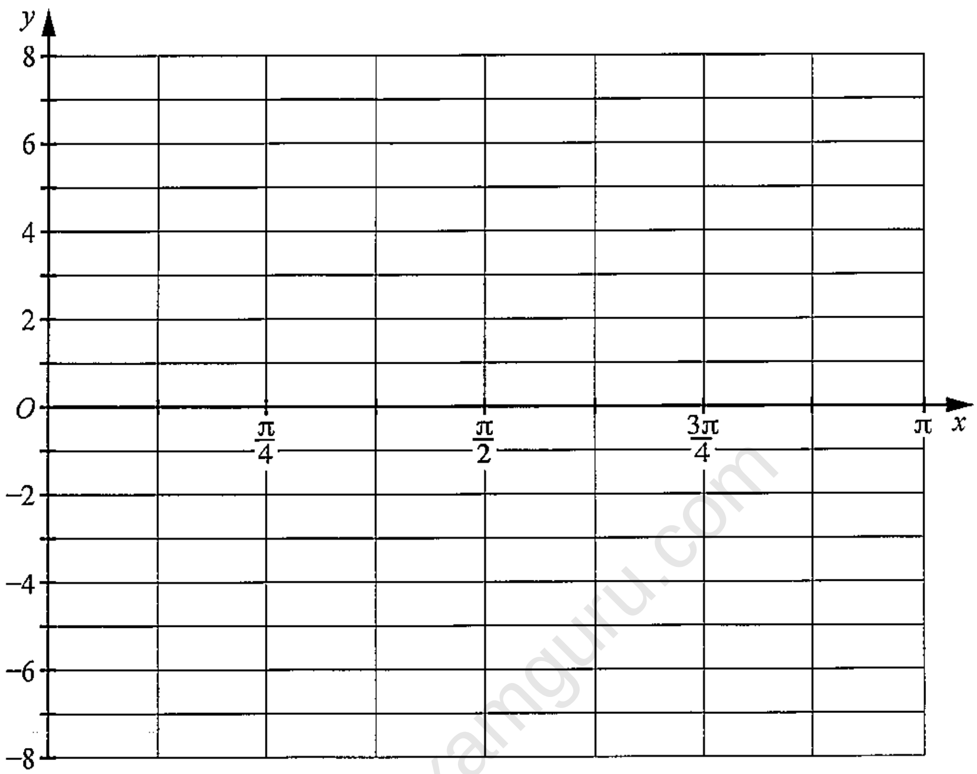
(b) **(i)** Write down the amplitude and period of $5\cos 4x - 3$. [2]

(ii) Write down the period of $4\tan 3x$. [1]

Oct Nov 2012 Code 12

6 **(i)** On the axes below sketch, for $0 \leqslant x \leqslant \pi$, the graphs of

$$y = \tan x \quad \text{and} \quad y = 1 + 3\sin 2x. \qquad [3]$$



Write down

(ii) the coordinates of the stationary points on the curve $y = 1 + 3\sin 2x$ for $0 \leqslant x \leqslant \pi$, [2]

(iii) the number of solutions of the equation $\tan x = 1 + 3\sin 2x$ for $0 \leqslant x \leqslant \pi$. [1]

Oct Nov 2012 Code 13

7 (i) Solve $\tan^2 x - 2\sec x + 1 = 0$ for $0° \leqslant x \leqslant 360°$. [4]

(ii) Solve $\cos^2 3y = 5\sin^2 3y$ for $0 \leqslant y \leqslant 2$ radians. [4]

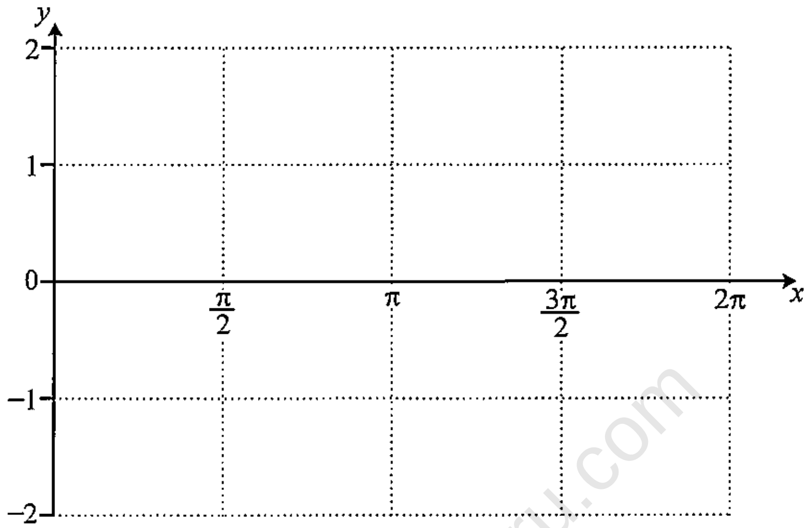
(iii) Solve $2\text{cosec}\left(z + \frac{\pi}{4}\right) = 5$ for $0 \leqslant z \leqslant 6$ radians. [4]

Oct Nov 2012 Code 13

8 On the axes below sketch, for $0 \leqslant x \leqslant 2\pi$, the graph of

(i) $y = \cos x - 1$, [2]

(ii) $y = \sin 2x$. [2]



(iii) State the number of solutions of the equation $\cos x - \sin 2x = 1$, for $0 \leqslant x \leqslant 2\pi$. [1]

May June 2013 Code 11,13

9 (a) Solve $2\sin\left(x+\frac{\pi}{3}\right)=-1$ for $0 \leqslant x \leqslant 2\pi$ radians. [4]

(b) Solve $\tan y - 2 = \cot y$ for $0° \leqslant y \leqslant 180°$. [6]

May June 2013 Code 11,13

10 Show that $(1-\cos\theta-\sin\theta)^2-2(1-\sin\theta)(1-\cos\theta)=0$. [3]

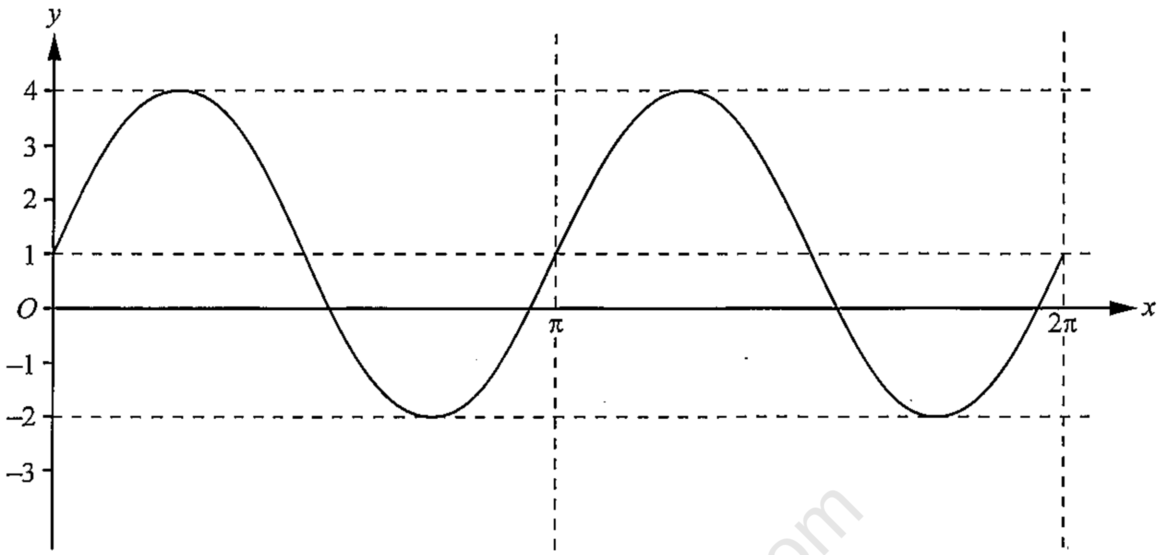
May June 2013 Code 12

11 **(a)** Solve $\cos 2x + 2\sec 2x + 3 = 0$ for $0° \leqslant x \leqslant 360°$. [5]

(b) Solve $2\sin^2\left(y - \frac{\pi}{6}\right) = 1$ for $0 \leqslant y \leqslant \pi$. [4]

May June 2013 Code 12

12 The diagram shows the graph of $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$, where a, b and c are positive integers.



State the value of a, of b and of c. [3]

$a =$ $b =$ $c =$

Oct Nov 2013 Code 11,12

13 Show that $\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$. [4]

Oct Nov 2013 Code 11,12

14 Show that $\tan^2\theta - \sin^2\theta = \sin^4\theta\sec^2\theta$. [4]

Oct Nov 2013 Code 13

15 **(a)** **(i)** Solve $6\sin^2 x = 5 + \cos x$ for $0° < x < 180°$. [4]

(ii) Hence, or otherwise, solve $6\cos^2 y = 5 + \sin y$ for $0° < y < 180°$. [3]

(b) Solve $4\cot^2 z - 3\cot z = 0$ for $0 < z < \pi$ radians. [4]

Oct Nov 2013 Code 13

16 Show that $\tan\theta + \dfrac{\cos\theta}{1+\sin\theta} = \sec\theta.$ [4]

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17 **a)** Solve $5\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 180°$. [4]

(b) Solve $2\cot^2 y + 3\operatorname{cosec} y = 0$ for $0° \leqslant y \leqslant 360°$. [4]

(c) Solve $3\cos(z + 1.2) = 2$ for $0 \leqslant z \leqslant 6$ radians. [4]

May June 2014 Code 11

18 Show that $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$ can be written in the form $p\sec A$, where p is an integer to be found. [4]

May June 2014 Code 12

19 **(a)** Solve $\tan^2 x + 5\tan x = 0$ for $0° \leqslant x \leqslant 180°$. [3]

(b) Solve $2\cos^2 y - \sin y - 1 = 0$ for $0° \leqslant y \leqslant 360°$. [4]

(c) Solve $\sec\left(2z - \frac{\pi}{6}\right) = 2$ for $0 \leqslant z \leqslant \pi$ radians. [4]

May June 2014 Code 12

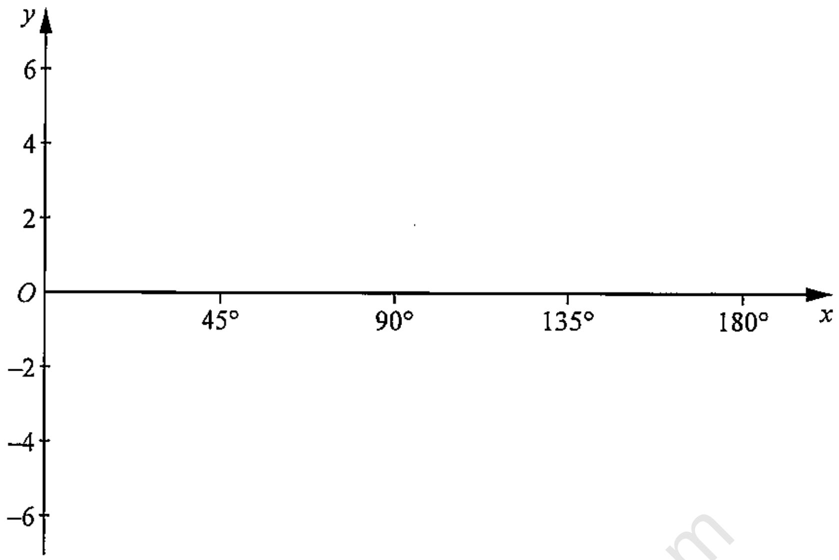
20 Solve

(i) $3\sin x\cos x = 2\cos x$ for $0° \leqslant x \leqslant 180°$, [4]

(ii) $10\sin^2 y + \cos y = 8$ for $0° \leqslant y \leqslant 360°$. [5]

May June 2014 Code 13

21 **(a)** On the axes below, sketch the curve $y = 3\cos 2x - 1$ for $0° \leqslant x \leqslant 180°$. [3]



(b) **(i)** State the amplitude of $1 - 4\sin 2x$. [1]

(ii) State the period of $5\tan 3x + 1$. [1]

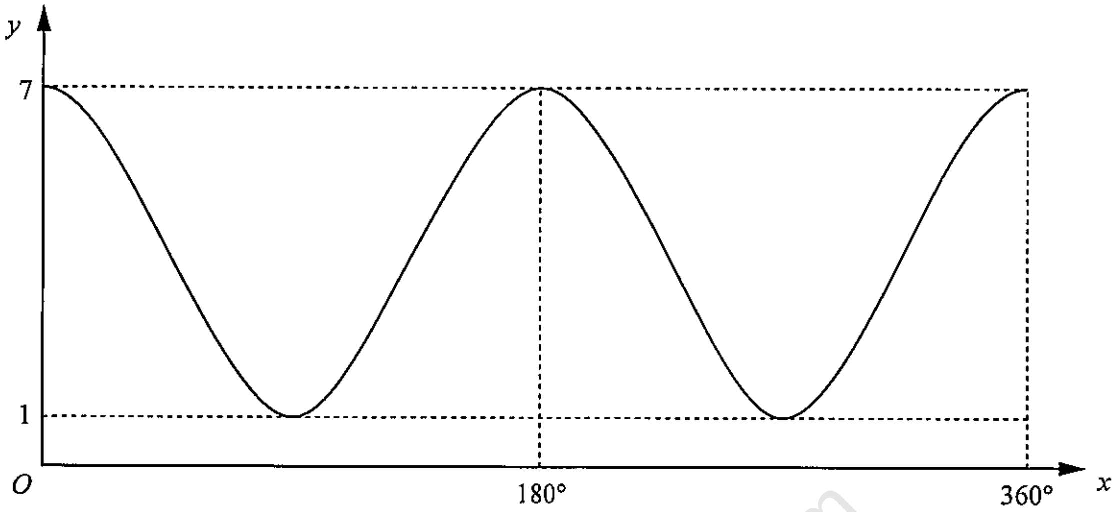
Oct Nov 2014 Code 11,12

22 **(a)** Solve $2\cos 3x = \cot 3x$ for $0° \leqslant x \leqslant 90°$. [5]

(b) Solve $\sec\left(y+\frac{\pi}{2}\right)=-2$ for $0 \leqslant y \leqslant \pi$ radians. [4]

Oct Nov 2014 Code 11,12

23 The diagram shows the graph of $y = a\cos bx + c$ for $0° \leqslant x \leqslant 360°$, where a, b and c are positive integers.



State the value of each of a, b and c. [3]

Oct Nov 2014 Code 13

24 **(a)** Solve $3\sin x + 5\cos x = 0$ for $0° \leqslant x \leqslant 360°$. [3]

(b) Solve $\operatorname{cosec}\left(3y + \frac{\pi}{4}\right) = 2$ for $0 \leqslant y \leqslant \pi$ radians. [5]

Oct Nov 2014 Code 13

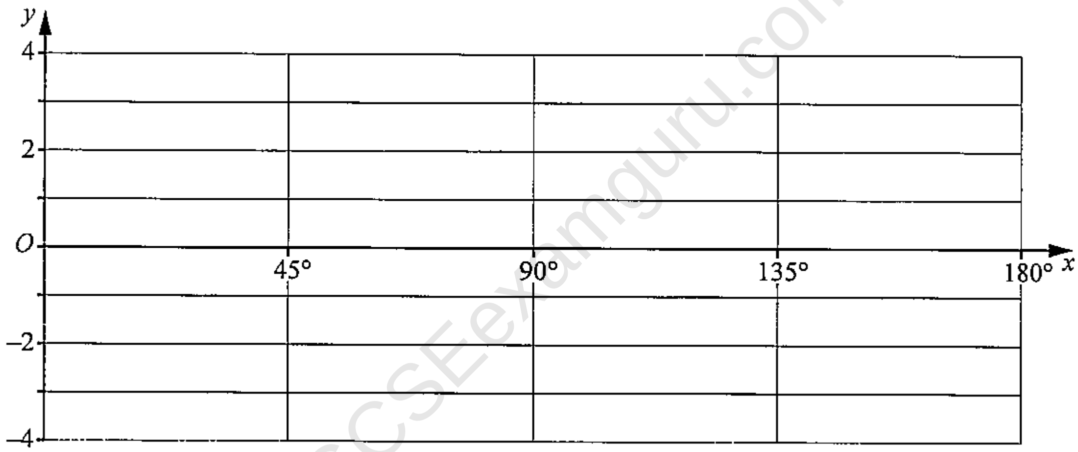
25 **(i)** State the period of $\sin 2x$. [1]

(ii) State the amplitude of $1 + 2\cos 3x$. [1]

(iii) On the axes below, sketch the graph of

(a) $y = \sin 2x$ for $0° \leqslant x \leqslant 180°$, [1]

(b) $y = 1 + 2\cos 3x$ for $0° \leqslant x \leqslant 180°$. [2]



(iv) State the number of solutions of $\sin 2x - 2\cos 3x = 1$ for $0° \leqslant x \leqslant 180°$. [1]

May June 2015 Code 11,13

26 **(a)** Solve $4\sin x = \operatorname{cosec} x$ for $0° \leqslant x \leqslant 360°$. [3]

(b) Solve $\tan^2 3y - 2\sec 3y - 2 = 0$ for $0° \leqslant y \leqslant 180°$. [6]

(c) Solve $\tan\left(z - \frac{\pi}{3}\right) = \sqrt{3}$ for $0 \leqslant z \leqslant 2\pi$ radians. [3]

May June 2015 Code 11,13

27 Show that $\frac{\tan\theta + \cot\theta}{\operatorname{cosec}\theta} = \sec\theta$. [4]

May June 2015 Code 12

28 **(a)** Solve $2\cos 3x = \sec 3x$ for $0° \leqslant x \leqslant 120°$. [3]

(b) Solve $3\operatorname{cosec}^2 y + 5\cot y - 5 = 0$ for $0° \leqslant y \leqslant 360°$. [5]

(c) Solve $2\sin\left(z + \frac{\pi}{3}\right) = 1$ for $0 \leqslant z \leqslant 2\pi$ radians. [4]

May June 2015 Code 12

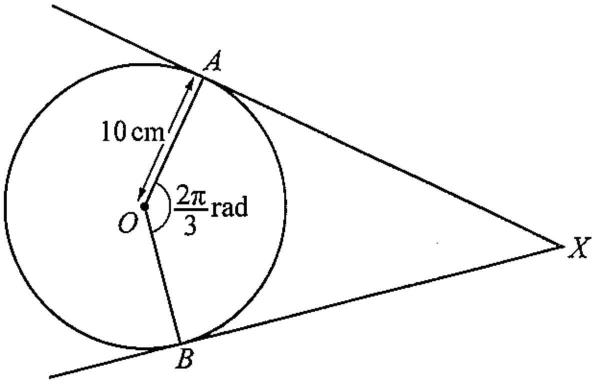
29 Show that $\sqrt{\sec^2\theta - 1} + \sqrt{\operatorname{cosec}^2\theta - 1} = \sec\theta \operatorname{cosec}\theta$. [5]

Paper 1 - Oct Nov 2015 Code 11

30 Solve $2\cos^2\left(3x - \frac{\pi}{4}\right) = 1$ for $0 \leqslant x \leqslant \frac{\pi}{3}$. [4]

Paper 1 - Oct Nov 2015 Code 13

1



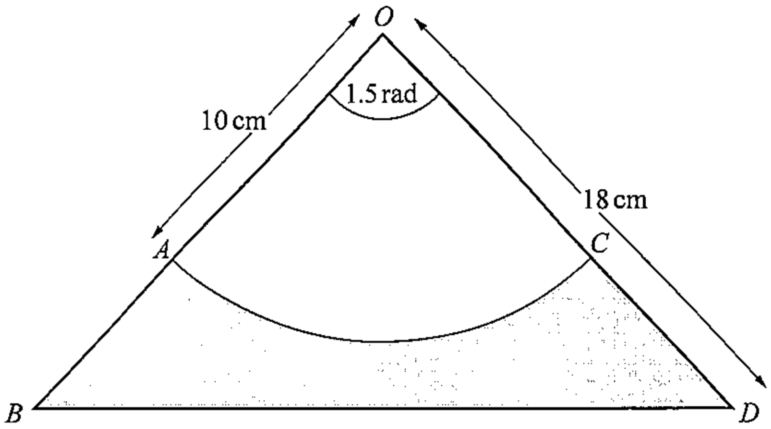
The figure shows a circle, centre O, with radius 10 cm. The lines XA and XB are tangents to the circle at A and B respectively, and angle AOB is $\frac{2\pi}{3}$ radians.

(i) Find the perimeter of the shaded region. [3]

(ii) Find the area of the shaded region. [4]

May June 2012 Code 12

2



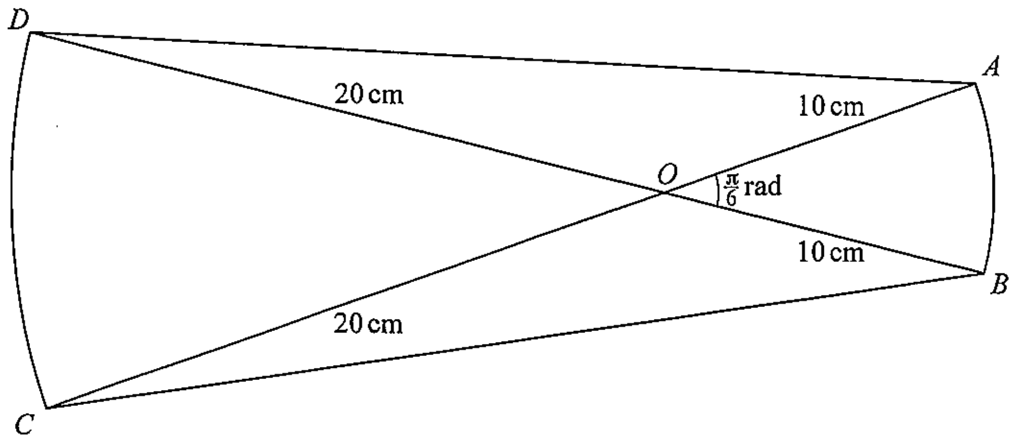
The diagram shows an isosceles triangle OBD in which $OB = OD = 18$ cm and angle $BOD = 1.5$ radians. An arc of the circle, centre O and radius 10 cm, meets OB at A and OD at C.

(i) Find the area of the shaded region. [3]

(ii) Find the perimeter of the shaded region. [4]

Oct Nov 2012 Code 12

3



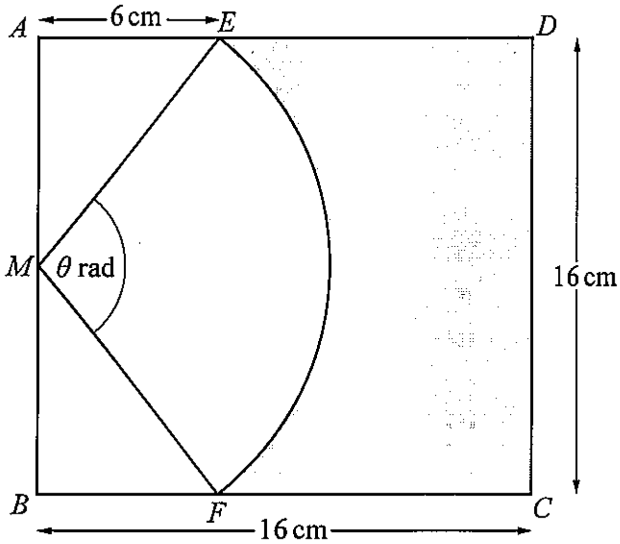
The diagram shows four straight lines, AD, BC, AC and BD. Lines AC and BD intersect at O such that angle AOB is $\frac{\pi}{6}$ radians. AB is an arc of the circle, centre O and radius 10 cm, and CD is an arc of the circle, centre O and radius 20 cm.

(i) Find the perimeter of $ABCD$. [4]

(ii) Find the area of $ABCD$. [4]

Oct Nov 2012 Code 13

4



The diagram shows a square *ABCD* of side 16 cm. *M* is the mid-point of *AB*. The points *E* and *F* are on *AD* and *BC* respectively such that AE = BF = 6 cm. *EF* is an arc of the circle centre *M*, such that angle *EMF* is θ radians.

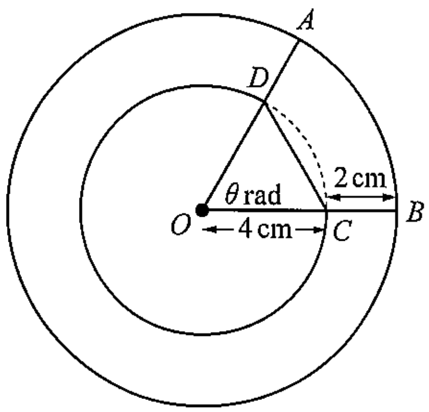
(i) Show that $\theta = 1.855$ radians, correct to 3 decimal places. [2]

(ii) Calculate the perimeter of the shaded region. [4]

(iii) Calculate the area of the shaded region. [3]

May June 2013 Code 11,13

5 The diagram shows two concentric circles, centre O, radii 4 cm and 6 cm. The points A and B lie on the larger circle and the points C and D lie on the smaller circle such that ODA and OCB are straight lines.



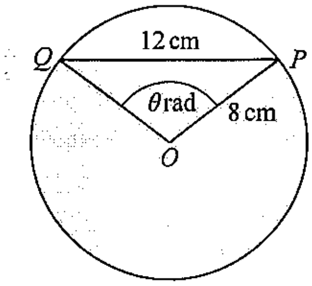
(i) Given that the area of triangle OCD is $7.5\,\text{cm}^2$, show that $\theta = 1.215$ radians, to 3 decimal places. [2]

(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [3]

Oct Nov 2013 Code 13

6 The diagram shows a circle, centre O, radius 8 cm. Points P and Q lie on the circle such that the chord $PQ = 12$ cm and angle $POQ = \theta$ radians.



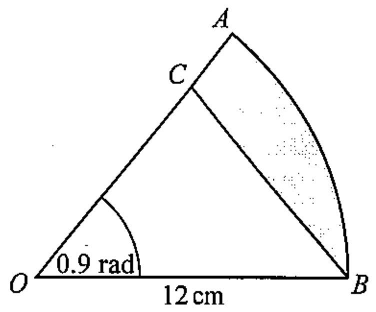
(i) Show that $\theta = 1.696$, correct to 3 decimal places. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

May June 2014 Code 12

7 The diagram shows a sector, AOB, of a circle centre O, radius 12 cm. Angle $AOB = 0.9$ radians. The point C lies on OA such that $OC = CB$.



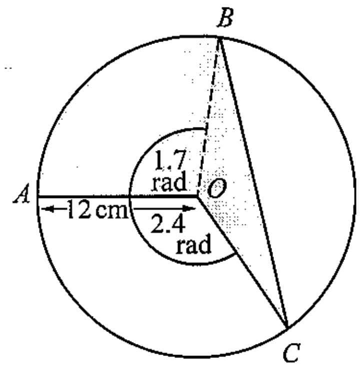
(i) Show that $OC = 9.65$ cm correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

Oct Nov 2014 Code 13

8



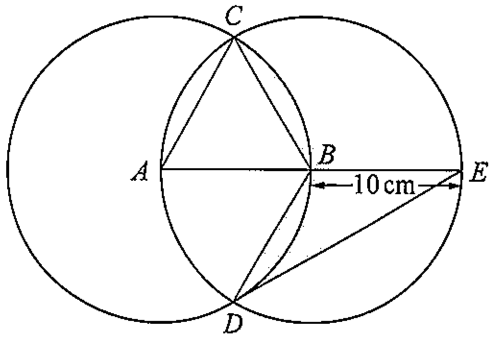
The diagram shows a circle, centre O, radius 12 cm. The points A, B and C lie on the circumference of this circle such that angle AOB is 1.7 radians and angle AOC is 2.4 radians.

(i) Find the area of the shaded region. [4]

(ii) Find the perimeter of the shaded region. [5]

May June 2015 Code 12

9



The diagram shows two circles, centres A and B, each of radius 10 cm. The point B lies on the circumference of the circle with centre A. The two circles intersect at the points C and D. The point E lies on the circumference of the circle centre B such that ABE is a diameter.

(i) Explain why triangle ABC is equilateral. [1]

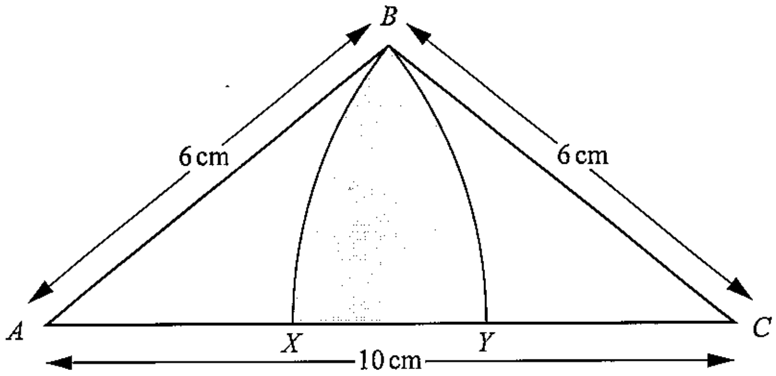
(ii) Write down, in terms of π, angle CBE. [1]

(iii) Find the perimeter of the shaded region. [5]

(iv) Find the area of the shaded region. [3]

Paper 1 - Oct Nov 2015 Code 11

10



The diagram shows an isosceles triangle ABC such that $AC = 10$ cm and $AB = BC = 6$ cm. BX is an arc of a circle, centre C, and BY is an arc of a circle, centre A.

(i) Show that angle $ABC = 1.970$ radians, correct to 3 decimal places. [2]

(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [3]

Paper 1 - Oct Nov 2015 Code 13

1 **(a)** Arrangements containing 5 different letters from the word AMPLITUDE are to be made. Find

(i) the number of 5-letter arrangements if there are no restrictions, [1]

(ii) the number of 5-letter arrangements which start with the letter A and end with the letter E. [1]

(b) Tickets for a concert are given out randomly to a class containing 20 students. No student is given more than one ticket. There are 15 tickets.

(i) Find the number of ways in which this can be done. [1]

There are 12 boys and 8 girls in the class. Find the number of different ways in which

(ii) 10 boys and 5 girls get tickets, [3]

(iii) all the boys get tickets. [1]

May June 2012 Code 11,13

2 A committee of 7 members is to be selected from 6 women and 9 men. Find the number of different committees that may be selected if

(i) there are no restrictions, [1]

(ii) the committee must consist of 2 women and 5 men, [2]

(iii) the committee must contain at least 1 woman. [3]

Oct Nov 2012 Code 13

3 A committee of 6 members is to be selected from 5 men and 9 women. Find the number of different committees that could be selected if

(i) there are no restrictions, [1]

(ii) there are exactly 3 men and 3 women on the committee, [2]

(iii) there is at least 1 man on the committee. [3]

May June 2013 Code 11,13

4 A 4-digit number is to be formed from the digits 1, 2, 5, 7, 8 and 9. Each digit may only be used once. Find the number of different 4-digit numbers that can be formed if

(i) there are no restrictions, [1]

(ii) the 4-digit numbers are divisible by 5, [2]

(iii) the 4-digit numbers are divisible by 5 and are greater than 7000. [2]

May June 2013 Code 12

5 (a) (i) Find how many different 4-digit numbers can be formed from the digits 1, 3, 5, 6, 8 and 9 if each digit may be used only once. [1]

(ii) Find how many of these 4-digit numbers are even. [1]

(b) A team of 6 people is to be selected from 8 men and 4 women. Find the number of different teams that can be selected if

(i) there are no restrictions, [1]

(ii) the team contains all 4 women, [1]

(iii) the team contains at least 4 men. [3]

Oct Nov 2013 Code 11

6 **(a)** How many even numbers less than 500 can be formed using the digits 1, 2, 3, 4 and 5? Each digit may be used only once in any number. [4]

(b) A committee of 8 people is to be chosen from 7 men and 5 women. Find the number of different committees that could be selected if

(i) the committee contains at least 3 men and at least 3 women, [4]

(ii) the oldest man or the oldest woman, but not both, must be included in the committee. [2]

May June 2014 Code 11

7 (a) (i) How many different 5-digit numbers can be formed using the digits 1, 2, 4, 5, 7 and 9 if no digit is repeated? [1]

(ii) How many of these numbers are even? [1]

(iii) How many of these numbers are less than 60 000 and even? [3]

(b) How many different groups of 6 children can be chosen from a class of 18 children if the class contains one set of twins who must not be separated? [3]

May June 2014 Code 12

8 **(a)** A 5-character password is to be chosen from the letters A, B, C, D, E and the digits 4, 5, 6, 7. Each letter or digit may be used only once. Find the number of different passwords that can be chosen if

(i) there are no restrictions, [1]

(ii) the password contains 2 letters followed by 3 digits. [2]

(b) A school has 3 concert tickets to give out at random to a class of 18 boys and 15 girls. Find the number of ways in which this can be done if

(i) there are no restrictions, [1]

(ii) 2 of the tickets are given to boys and 1 ticket is given to a girl, [2]

(iii) at least 1 boy gets a ticket. [2]

May June 2014 Code 13

9 (a) (i) Find how many different 4-digit numbers can be formed using the digits 1, 2, 3, 4, 5 and 6 if no digit is repeated. [1]

(ii) How many of the 4-digit numbers found in part (i) are greater than 6000? [1]

(iii) How many of the 4-digit numbers found in part (i) are greater than 6000 and are odd? [1]

(b) A quiz team of 10 players is to be chosen from a class of 8 boys and 12 girls.

(i) Find the number of different teams that can be chosen if the team has to have equal numbers of girls and boys. [3]

(ii) Find the number of different teams that can be chosen if the team has to include the youngest and oldest boy and the youngest and oldest girl. [2]

Oct Nov 2014 Code 11,12

10 **(a)** A security code is to be chosen using 6 of the following:

- the letters A, B and C
- the numbers 2, 3 and 5
- the symbols * and $.

None of the above may be used more than once. Find the number of different security codes that may be chosen if

(i) there are no restrictions, [1]

(ii) the security code starts with a letter and finishes with a symbol, [2]

(iii) the two symbols are next to each other in the security code. [3]

(b) Two teams, each of 4 students, are to be selected from a class of 8 boys and 6 girls. Find the number of different ways the two teams may be selected if

(i) there are no restrictions, [2]

(ii) one team is to contain boys only and the other team is to contain girls only. [2]

May June 2015 Code 12

11 (a) 6 books are to be chosen from 8 different books.

(i) Find the number of different selections of 6 books that could be made. [1]

A clock is to be displayed on a shelf with 3 of the 8 different books on each side of it. Find the number of ways this can be done if

(ii) there are no restrictions on the choice of books, [1]

(iii) 3 of the 8 books are music books which have to be kept together. [2]

(b) A team of 6 tennis players is to be chosen from 10 tennis players consisting of 7 men and 3 women. Find the number of different teams that could be chosen if the team must include at least 1 woman. [3]

Paper 1 - Oct Nov 2015 Code 11

12 **(a)** Five different books are to be arranged on a shelf. There are 2 Mathematics books and 3 History books. Find the number of different arrangements of books if

(i) the Mathematics books are next to each other, [2]

(ii) the Mathematics books are not next to each other. [2]

(b) To compete in a quiz, a team of 5 is to be chosen from a group of 9 men and 6 women. Find the number of different teams that can be chosen if

(i) there are no restrictions, [1]

(ii) at least two men must be on the team. [3]

Paper 1 - Oct Nov 2015 Code 13

1 Find the values of the positive constants p and q such that, in the binomial expansion of $(p + qx)^{10}$, the coefficient of x^5 is 252 and the coefficient of x^3 is 6 times the coefficient of x^2.

[8]

May June 2012 Code 11,13

2 **(i)** Find the first 3 terms, in descending powers of x, in the expansion of $\left(x + \frac{2}{x^2}\right)^6$. [3]

(ii) Hence find the term independent of x in the expansion of $\left(2 - \frac{4}{x^3}\right)\left(x + \frac{2}{x^2}\right)^6$. [2]

Oct Nov 2012 Code 11

3 In the expansion of $(p+x)^6$, where p is a positive integer, the coefficient of x^2 is equal to 1.5 times the coefficient of x^3.

(i) Find the value of p. [4]

(ii) Use your value of p to find the term independent of x in the expansion of $(p+x)^6\left(1-\frac{1}{x}\right)^2$. [3]

Oct Nov 2012 Code 13

4 **(i)** Given that n is a positive integer, find the first 3 terms in the expansion of $\left(1+\frac{1}{2}x\right)^n$ in ascending powers of x. [2]

(ii) Given that the coefficient of x^2 in the expansion of $(1-x)\left(1+\frac{1}{2}x\right)^n$ is $\frac{25}{4}$, find the value of n. [5]

May June 2013 Code 12

5 The coefficient of x^2 in the expansion of $(2 + px)^6$ is 60.

(i) Find the value of the positive constant p. [3]

(ii) Using your value of p, find the coefficient of x^2 in the expansion of $(3 - x)(2 + px)^6$. [3]

Oct Nov 2013 Code 13

6 (i) The first three terms in the expansion of $(2-5x)^6$, in ascending powers of x, are $p+qx+rx^2$. Find the value of each of the integers p, q and r. [3]

(ii) In the expansion of $(2-5x)^6(a+bx)^3$, the constant term is equal to 512 and the coefficient of x is zero. Find the value of each of the constants a and b. [4]

May June 2014 Code 13

7 **(i)** Given that the coefficient of x^2 in the expansion of $(2+px)^6$ is 60, find the value of the positive constant p. [3]

(ii) Using your value of p, find the coefficient of x^2 in the expansion of $(3-x)(2+px)^6$. [3]

Oct Nov 2014 Code 11,12

8 **(a)** Given that the first 3 terms in the expansion of $(5 - qx)^p$ are $625 - 1500x + rx^2$, find the value of each of the integers p, q and r. [5]

(b) Find the value of the term that is independent of x in the expansion of $\left(2x + \frac{1}{4x^3}\right)^{12}$. [3]

Oct Nov 2014 Code 13

9 (i) Find the first 4 terms in the expansion of $(2+x^2)^6$ in ascending powers of x. [3]

(ii) Find the term independent of x in the expansion of $(2+x^2)^6\left(1-\dfrac{3}{x^2}\right)^2$. [3]

May June 2015 Code 11,13

10 **(a)** Given that the first 4 terms in the expansion of $(2+kx)^8$ are $256+256x+px^2+qx^3$, find the value of k, of p and of q. [3]

(b) Find the term that is independent of x in the expansion of $\left(x-\frac{2}{x^2}\right)^9$. [3]

Paper 1 - Oct Nov 2015 Code 13

1 **(i)** Find the equation of the tangent to the curve $y = x^3 + 2x^2 - 3x + 4$ at the point where the curve crosses the y-axis. [4]

(ii) Find the coordinates of the point where this tangent meets the curve again. [3]

May June 2012 Code11,13

2 Variables x and y are such that $y = e^{2x} + e^{-2x}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) By using the substitution $u = e^{2x}$, find the value of y when $\frac{dy}{dx} = 3$. [4]

(iii) Given that x is decreasing at the rate of 0.5 units s^{-1}, find the corresponding rate of change of y when $x = 1$. [3]

May June 2012 Code11,13

3 Differentiate the following with respect to x.

(i) $(2 - x^2)\ln(3x + 1)$ [3]

(ii) $\dfrac{4 - \tan 2x}{5x}$ [3]

May June 2012 Code12

4 Given that $y = \dfrac{x^2}{\cos 4x}$, find

(i) $\dfrac{dy}{dx}$, [3]

(ii) the approximate change in y when x increases from $\dfrac{\pi}{4}$ to $\dfrac{\pi}{4} + p$, where p is small. [2]

Oct Nov 2012 Code11

5 A curve is such that $y = \dfrac{Ax^2 + B}{x^2 - 2}$, where A and B are constants.

(i) Show that $\dfrac{dy}{dx} = -\dfrac{2x(2A + B)}{(x^2 - 2)^2}$. [4]

It is given that $y = -3$ and $\dfrac{dy}{dx} = -10$ when $x = 1$.

(ii) Find the value of A and of B. [3]

(iii) Using your values of A and B, find the coordinates of the stationary point on the curve, and determine the nature of this stationary point. [4]

Oct Nov 2012 Code12

6 The rate of change of a variable x with respect to time t is $4\cos^2 t$.

(i) Find the rate of change of x with respect to t when $t = \frac{\pi}{6}$. [1]

The rate of change of a variable y with respect to time t is $3\sin t$.

(ii) Using your result from part **(i)**, find the rate of change of y with respect to x when $t = \frac{\pi}{6}$. [3]

Oct Nov 2012 Code13

7 The tangent to the curve $y = 5e^x + 3e^{-x}$ at the point where $x = \ln\frac{3}{5}$, meets the x-axis at the point P.

(i) Find the coordinates of P. [5]

The area of the region enclosed by the curve $y = 5e^x + 3e^{-x}$, the y-axis, the positive x-axis and the line $x = a$ is 12 square units.

(ii) Show that $5e^{2a} - 14e^a - 3 = 0$. [3]

(iii) Hence find the value of a. [3]

Oct Nov 2012 Code13

8 The point A, whose x-coordinate is 2, lies on the curve with equation $y = x^3 - 4x^2 + x + 1$.

(i) Find the equation of the tangent to the curve at A. [4]

This tangent meets the curve again at the point B.

(ii) Find the coordinates of B. [4]

(iii) Find the equation of the perpendicular bisector of the line AB. [4]

May June 2013 Code11,13

9 The normal to the curve $y + 2 = 3\tan x$, at the point on the curve where $x = \frac{3\pi}{4}$, cuts the y-axis at the point P. Find the coordinates of P. [6]

May June 2013 Code 11

10 A curve has equation $y = \frac{e^{2x}}{(x+3)^2}$.

(i) Show that $\frac{dy}{dx} = \frac{Ae^{2x}(x+2)}{(x+3)^3}$, where A is a constant to be found. [4]

(ii) Find the exact coordinates of the point on the curve where $\frac{dy}{dx} = 0$. [2]

Oct Nov 2013 Code13

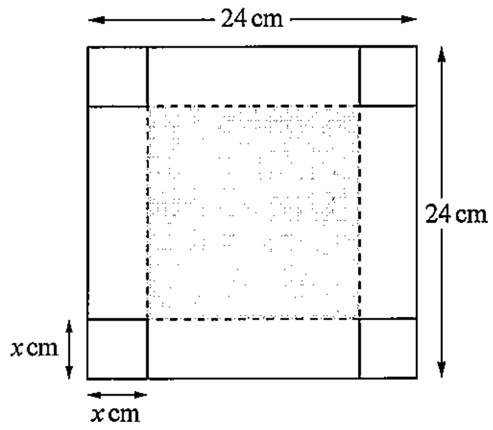
11 A solid circular cylinder has a base radius of r cm and a volume of $4000\,\text{cm}^3$.

(i) Show that the total surface area, $A\,\text{cm}^2$, of the cylinder is given by $A = \frac{8000}{r} + 2\pi r^2$. [3]

(ii) Given that r can vary, find the minimum total surface area of the cylinder, justifying that this area is a minimum. [6]

May June 2014 Code12

12 The diagram shows a thin square sheet of metal measuring 24 cm by 24 cm. A square of side x cm is cut off from each corner. The remainder is then folded to form an open box, x cm deep, whose square base is shown shaded in the diagram.



(i) Show that the volume, $V\text{cm}^3$, of the box is given by $V = 4x^3 - 96x^2 + 576x$. [2]

(ii) Given that x can vary, find the maximum volume of the box. [4]

May June 2014 Code13

13 Find the equation of the normal to the curve $y = x(x^2 - 12)^{\frac{1}{3}}$ at the point on the curve where $x = 2$. [6]

May June 2014 Code13

14 **(i)** Find the equation of the tangent to the curve $y = x^3 - \ln x$ at the point on the curve where $x = 1$. [4]

(ii) Show that this tangent bisects the line joining the points $(-2, 16)$ and $(12, 2)$. [2]

Oct Nov 2014 Code11,12

15 The point A, where $x = 0$, lies on the curve $y = \frac{\ln(4x^2+3)}{x-1}$. The normal to the curve at A meets the x-axis at the point B.

(i) Find the equation of this normal. [7]

(ii) Find the area of the triangle AOB, where O is the origin. [2]

May June 2015 Code11,13

16 A curve has equation $y = 4x + 3\cos 2x$. The normal to the curve at the point where $x = \frac{\pi}{4}$ meets the x- and y-axes at the points A and B respectively. Find the exact area of the triangle AOB, where O is the origin. [8]

May June 2015 Code12

17 Variables x and y are such that $y = (x - 3)\ln(2x^2 + 1)$.

(i) Find the value of $\frac{dy}{dx}$ when $x = 2$. [4]

(ii) Hence find the approximate change in y when x changes from 2 to 2.03. [2]

Paper 1 - Oct Nov 2015 Code 11

18 Find the equation of the tangent to the curve $y = \frac{2x - 1}{\sqrt{x^2 + 5}}$ at the point where $x = 2$. [7]

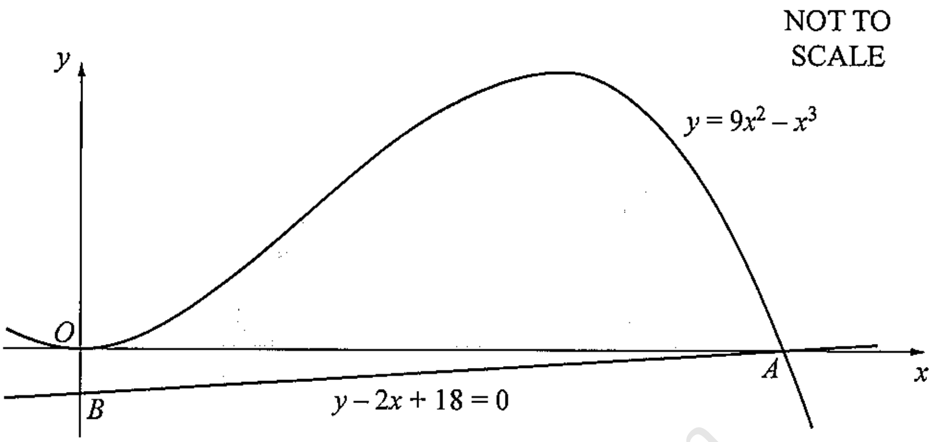
Paper 1 - Oct Nov 2015 Code 11

19 Find the equation of the normal to the curve $y = 5\tan x - 3$ at the point where $x = \frac{\pi}{4}$. [5]

Paper 1 - Oct Nov 2015 Code 13

1

The diagram shows part of the curve $y = 9x^2 - x^3$, which meets the x-axis at the origin O and at the point A. The line $y - 2x + 18 = 0$ passes through A and meets the y-axis at the point B.



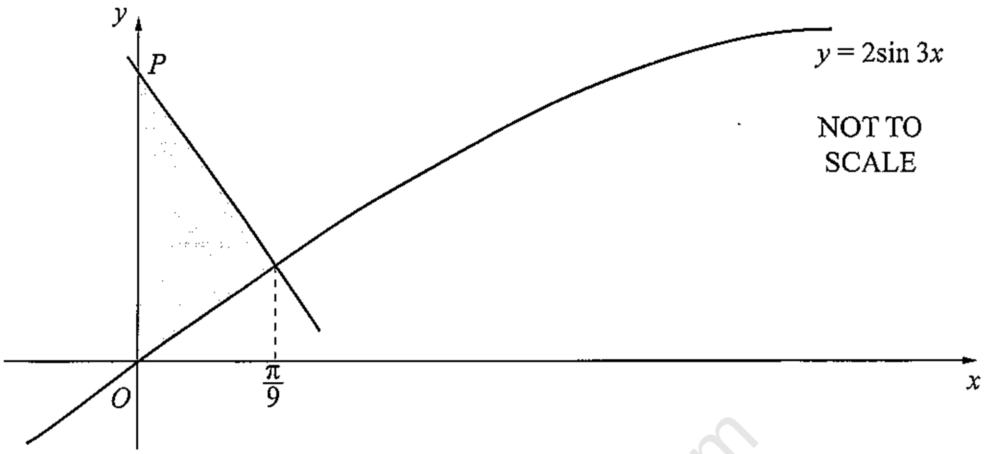
(i) Show that, for $x \geqslant 0$, $9x^2 - x^3 \leqslant 108$. [4]

(ii) Find the area of the shaded region bounded by the curve, the line AB and the y-axis. [6]

May June 2012 Code 11,13

2

The diagram shows part of the curve $y = 2\sin 3x$. The normal to the curve $y = 2\sin 3x$ at the point where $x = \frac{\pi}{9}$ meets the y-axis at the point P.



(i) Find the coordinates of P. [5]

(ii) Find the area of the shaded region bounded by the curve, the normal and the y-axis. [5]

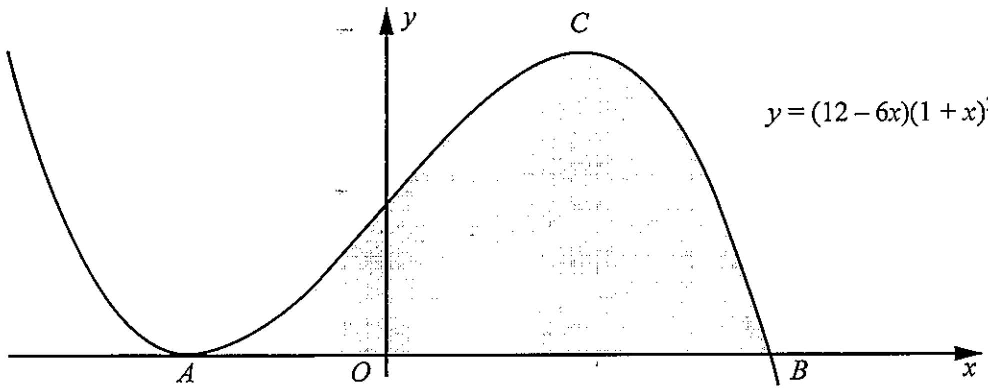
May June 2012 Code 11,13

3 **(i)** Find $\int \sqrt{7x-5}\,dx$. [3]

(ii) Hence evaluate $\int_2^3 \sqrt{7x-5}\,dx$. [2]

May June 2012 Code 12

4



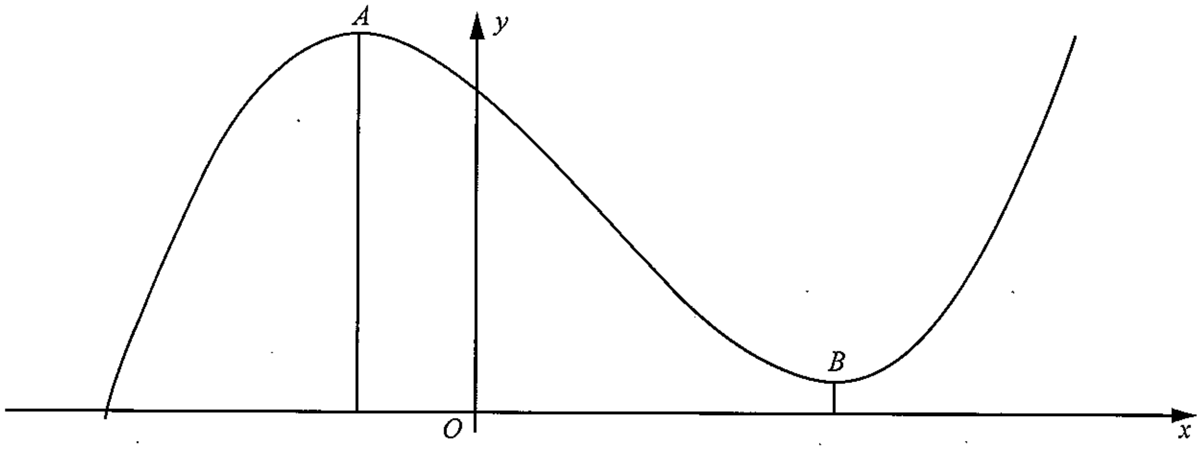
The diagram shows part of the graph of $y = (12 - 6x)(1 + x)^2$, which meets the x-axis at the points A and B. The point C is the maximum point of the curve.

(i) Find the coordinates of each of A, B and C. [6]

(ii) Find the area of the shaded region. [5]

Oct Nov 2012 Code 11

5



The diagram shows part of a curve such that $\frac{dy}{dx} = 3x^2 - 6x - 9$. Points A and B are stationary points of the curve and lines from A and B are drawn perpendicular to the x-axis. Given that the curve passes through the point (0, 30), find

(i) the equation of the curve, [4]

(ii) the x-coordinate of A and of B, [3]

(iii) the area of the shaded region. [4]

Oct Nov 2012 Code 11

6

A curve is such that $y = \dfrac{5x^2}{1 + x^2}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{kx}{(1 + x^2)^2}$, where k is an integer to be found. [4]

(ii) Find the coordinates of the stationary point on the curve and determine the nature of this stationary point. [3]

(iii) By using your result from part (i), find $\displaystyle\int \frac{x}{(1 + x^2)^2}\,dx$ and hence evaluate $\displaystyle\int_{-1}^{2} \frac{x}{(1 + x^2)^2}\,dx$. [4]

Oct Nov 2012 Code 12

7

(i) Given that $y = \dfrac{3e^{2x}}{1 + e^{2x}}$, show that $\dfrac{dy}{dx} = \dfrac{Ae^{2x}}{(1 + e^{2x})^2}$, where A is a constant to be found. [4]

(ii) Find the equation of the tangent to the curve $y = \dfrac{3e^{2x}}{1 + e^{2x}}$ at the point where the curve crosses the y-axis. [3]

(iii) Using your result from part **(i)**, find $\displaystyle\int \frac{e^{2x}}{(1 + e^{2x})^2}\,dx$ and hence evaluate $\displaystyle\int_0^{\ln 3} \frac{e^{2x}}{(1 + e^{2x})^2}\,dx$. [4]

Oct Nov 2012 Code 13

8 **(i)** Find $\int\left(1-\frac{6}{x^2}\right)\mathrm{d}x$. [2]

(ii) Hence find the value of the positive constant k for which $\int_k^{3k}\left(1-\frac{6}{x^2}\right)\mathrm{d}x = 2$. [4]

May June 2013 Code 11,13

9 (a) (i) Find $\int \sqrt{2x-5}\,dx$. [2]

(ii) Hence evaluate $\int_3^{15} \sqrt{2x-5}\,dx$. [2]

(b) (i) Find $\frac{d}{dx}(x^3 \ln x)$. [2]

(ii) Hence find $\int x^2 \ln x\,dx$. [3]

May June 2013 Code 12

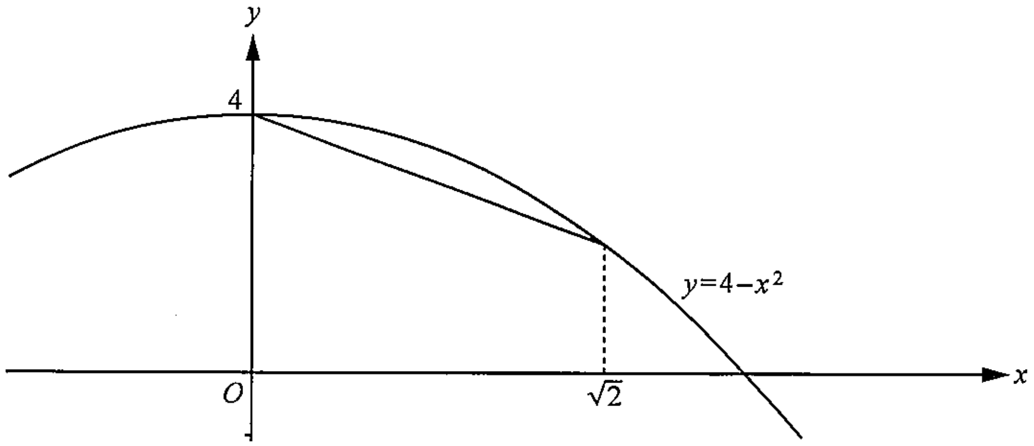
10 **(i)** Find $\int (9 + \sin 3x)\,dx$. [3]

(ii) Hence show that $\int_{\frac{\pi}{9}}^{\pi} (9 + \sin 3x)\,dx = a\pi + b$, where a and b are constants to be found. [3]

Oct Nov 2013 Code 11,12

11 **Do not use a calculator in this question.**

The diagram shows part of the curve $y = 4 - x^2$.



Show that the area of the shaded region can be written in the form $\frac{\sqrt{2}}{p}$, where p is an integer to be found. [6]

Oct Nov 2013 Code 13

12 (i) Given that $\int_0^k \left(2e^{2x} - \frac{5}{2}e^{-2x}\right) dx = \frac{3}{4}$, where k is a constant, show that

$$4e^{4k} - 12e^{2k} + 5 = 0.$$ [5]

(ii) Using a substitution of $y = e^{2k}$, or otherwise, find the possible values of k. [4]

Oct Nov 2013 Code 13

13 **(i)** Given that $y = e^{x^2}$, find $\frac{dy}{dx}$. [2]

(ii) Use your answer to part **(i)** to find $\int xe^{x^2}dx$. [2]

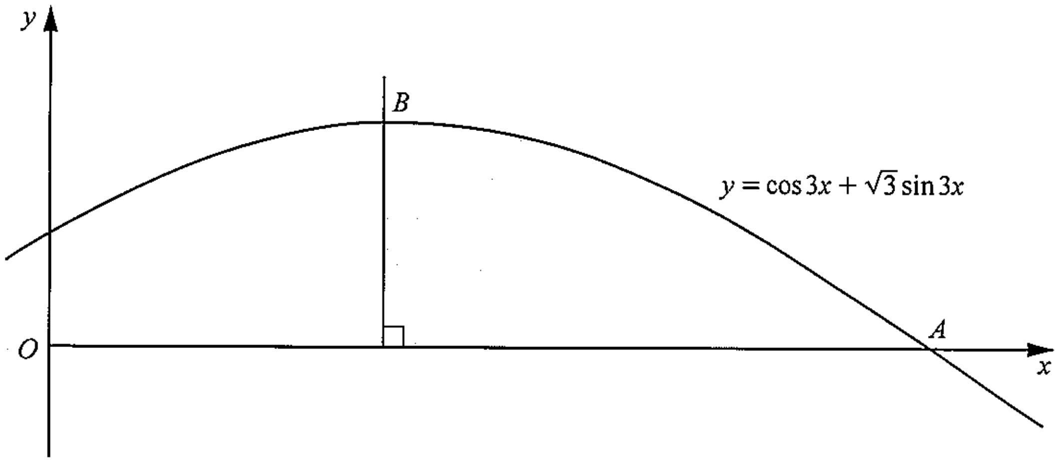
(iii) Hence evaluate $\int_0^2 xe^{x^2}dx$. [2]

May June 2014 Code 11

14 The region enclosed by the curve $y = 2\sin 3x$, the x-axis and the line $x = a$, where $0 < a < 1$ radian, lies entirely above the x-axis. Given that the area of this region is $\frac{1}{3}$ square unit, find the value of a. [6]

May June 2014 Code 12

15 The diagram shows the graph of $y = \cos 3x + \sqrt{3}\sin 3x$, which crosses the x-axis at A and has a maximum point at B.



(i) Find the x-coordinate of A. [3]

(ii) Find $\frac{dy}{dx}$ and hence find the x-coordinate of B. [4]

(iii) Showing all your working, find the area of the shaded region bounded by the curve, the x-axis and the line through B parallel to the y-axis. [5]

May June 2014 Code 13

16 A curve is such that $\frac{dy}{dx} = \frac{2}{\sqrt{x+3}}$ for $x > -3$. The curve passes through the point $(6, 10)$.

(i) Find the equation of the curve. [4]

(ii) Find the x-coordinate of the point on the curve where $y = 6$. [1]

Oct Nov 2014 Code 11,12

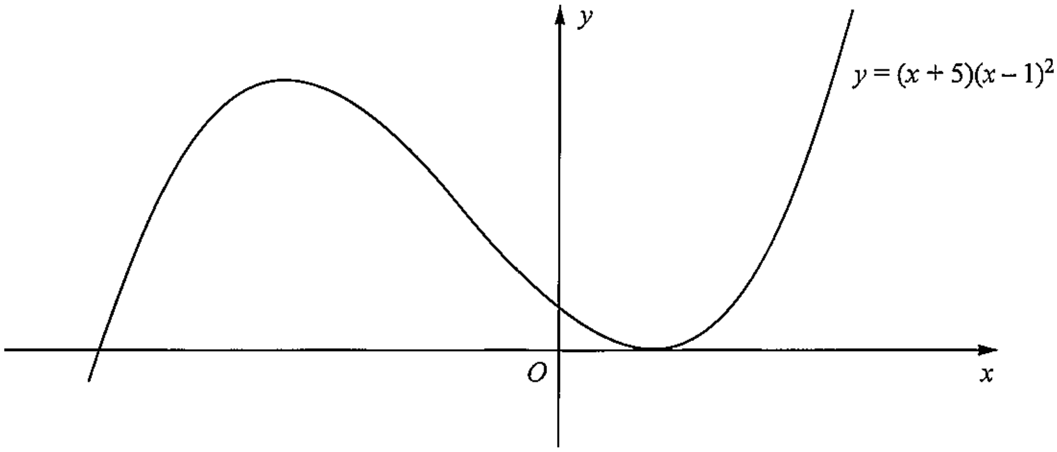
17 **(i)** Given that $f(x) = x\ln x^3$, show that $f'(x) = 3(1 + \ln x)$. [3]

(ii) Hence find $\int (1 + \ln x)\,dx$. [2]

(iii) Hence find $\int_1^2 \ln x\,dx$ in the form $p + \ln q$, where p and q are integers. [3]

Oct Nov 2014 Code 13

18 The diagram shows part of the curve $y = (x + 5)(x - 1)^2$.



(i) Find the x-coordinates of the stationary points of the curve. [5]

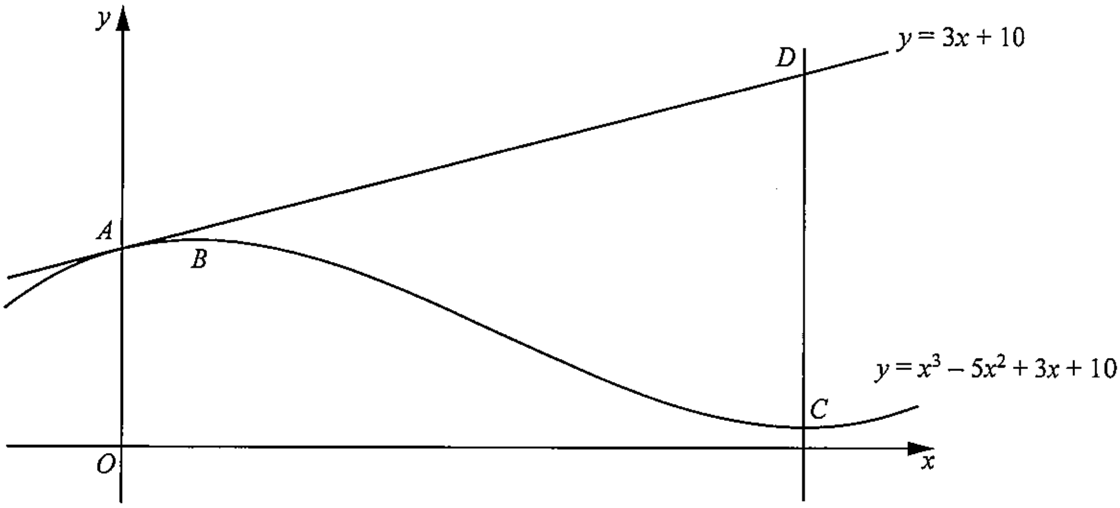
(ii) Find $\int (x + 5)(x - 1)^2 \, dx$. [3]

(iii) Hence find the area enclosed by the curve and the x-axis. [2]

(iv) Find the set of positive values of k for which the equation $(x + 5)(x - 1)^2 = k$ has only one real solution. [2]

Oct Nov 2014 Code 13

19



The diagram shows parts of the line $y = 3x + 10$ and the curve $y = x^3 - 5x^2 + 3x + 10$. The line and the curve both pass through the point A on the y-axis. The curve has a maximum at the point B and a minimum at the point C. The line through C, parallel to the y-axis, intersects the line $y = 3x + 10$ at the point D.

(i) Show that the line AD is a tangent to the curve at A. [2]

(ii) Find the x-coordinate of B and of C. [3]

(iii) Find the area of the shaded region $ABCD$, showing all your working. [5]

May June 2015 Code 11,13

20 **(i)** Find $\int(10e^{2x}+e^{-2x})dx$. [2]

(ii) Hence find $\int_{-k}^{k}(10e^{2x}+e^{-2x})dx$ in terms of the constant k. [2]

(iii) Given that $\int_{-k}^{k}(10e^{2x}+e^{-2x})dx=-60$, show that $11e^{2k}-11e^{-2k}+120=0$. [2]

(iv) Using a substitution of $y=e^{2k}$ or otherwise, find the value of k in the form $a\ln b$, where a and b are constants. [3]

May June 2015 Code 12

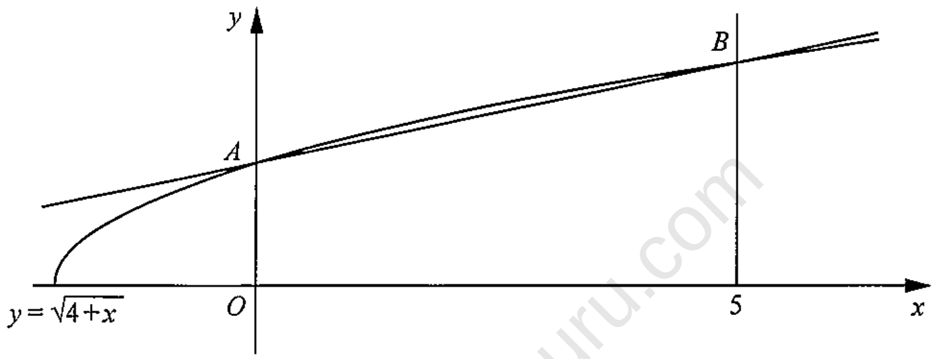
21 A curve, showing the relationship between two variables x and y, passes through the point $P(-1, 3)$. The curve has a gradient of 2 at P. Given that $\frac{d^2y}{dx^2} = -5$, find the equation of the curve. [4]

Paper 1 - Oct Nov 2015 Code 11

22 **You are not allowed to use a calculator in this question.**

(i) Find $\int \sqrt{4+x}\,dx$. [2]

(ii)



The diagram shows the graph of $y = \sqrt{4+x}$, which meets the y-axis at the point A and the line $x = 5$ at the point B. Using your answer to part **(i)**, find the area of the region enclosed by the curve and the straight line AB. [5]

Paper 1 - Oct Nov 2015 Code 11

23 A curve, showing the relationship between two variables x and y, is such that $\frac{d^2y}{dx^2} = 6\cos 3x$. Given that the curve has a gradient of $4\sqrt{3}$ at the point $\left(\frac{\pi}{9}, -\frac{1}{3}\right)$, find the equation of the curve. [6]

Paper 1 - Oct Nov 2015 Code 13

1 A particle P moves along the x-axis such that its distance, x m, from the origin O at time t s is given by $x = \frac{t}{t^2 + 1}$ for $t \geqslant 0$.

(i) Find the greatest distance of P from O. [4]

(ii) Find the acceleration of P at the instant when P is at its greatest distance from O. [3]

Oct Nov 2012 Code 13

2 A particle P moves in a straight line such that, t s after leaving a point O, its velocity $v\,\text{m}\,\text{s}^{-1}$ is given by $v = 36t - 3t^2$ for $t \geqslant 0$.

(i) Find the value of t when the velocity of P stops increasing. [2]

(ii) Find the value of t when P comes to instantaneous rest. [2]

(iii) Find the distance of P from O when P is at instantaneous rest. [3]

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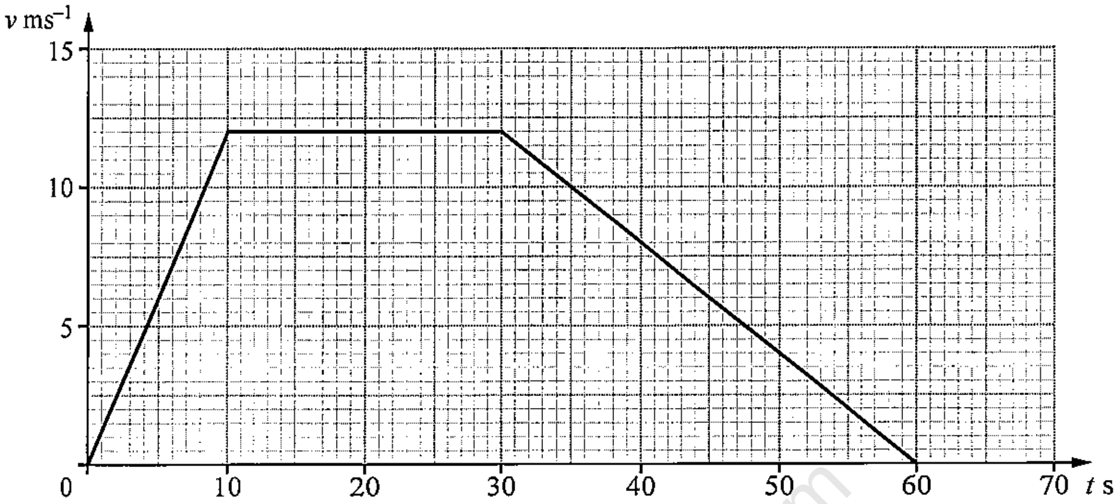
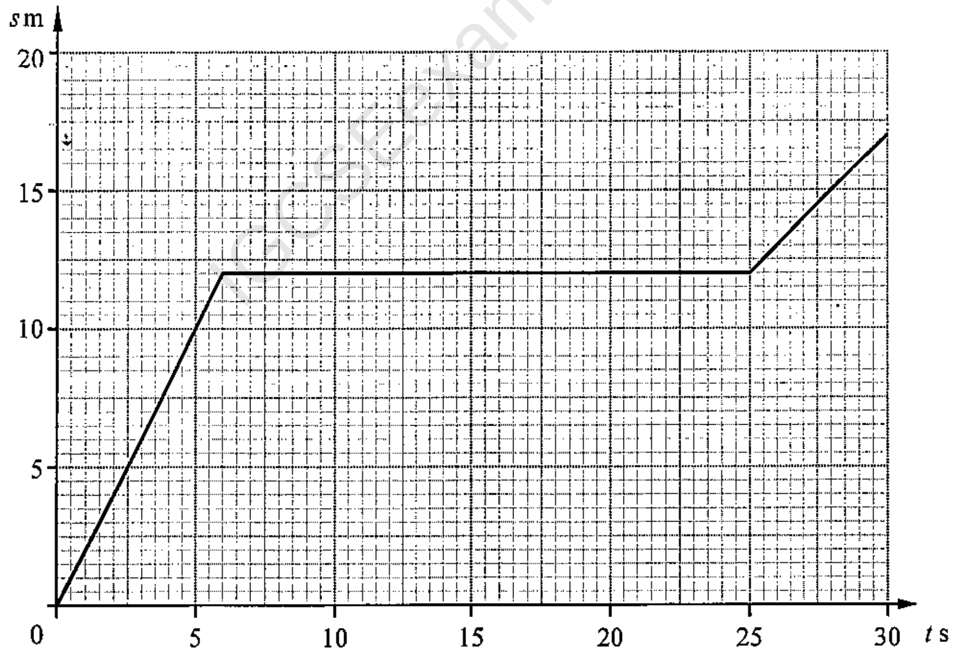
3 (a) The diagram shows the velocity-time graph of a particle P moving in a straight line with velocity v ms^{-1} at time t s after leaving a fixed point.

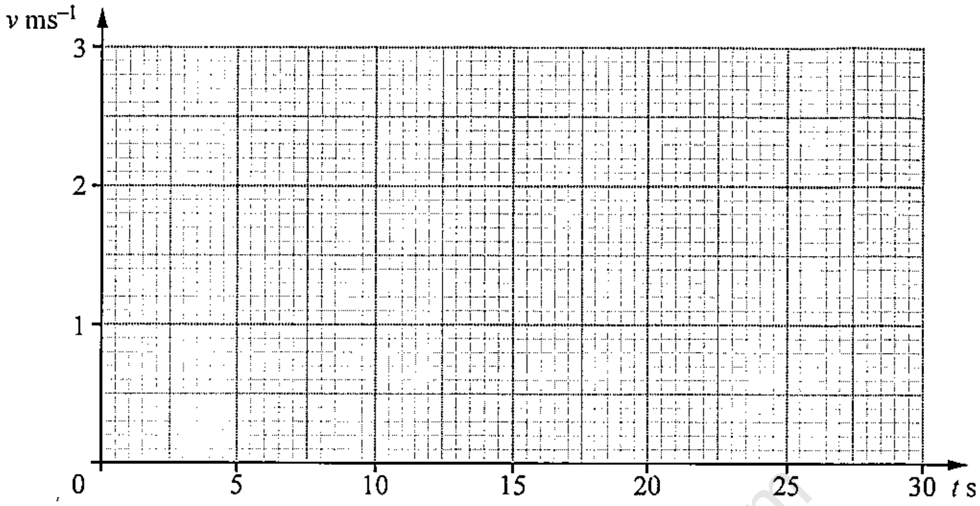


Find the distance travelled by the particle P. [2]

(b) The diagram shows the displacement-time graph of a particle Q moving in a straight line with displacement s m from a fixed point at time t s.



On the axes below, plot the corresponding velocity-time graph for the particle Q. [3]



(c) The displacement s m of a particle R, which is moving in a straight line, from a fixed point at time t s is given by $s = 4t - 16\ln(t+1) + 13$.

(i) Find the value of t for which the particle R is instantaneously at rest. [3]

(ii) Find the value of t for which the acceleration of the particle R is 0.25ms^{-2}. [2]

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4 A particle moves in a straight line such that, t s after passing through a fixed point O, its velocity, $v\,\text{ms}^{-1}$, is given by $v = 5 - 4e^{-2t}$.

(i) Find the velocity of the particle at O. [1]

(ii) Find the value of t when the acceleration of the particle is $6\,\text{ms}^{-2}$. [3]

(iii) Find the distance of the particle from O when $t = 1.5$. [5]

(iv) Explain why the particle does not return to O. [1]

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5 A particle moves in a straight line such that its displacement, x m, from a fixed point O after t s, is given by $x = 10\ln(t^2 + 4) - 4t$.

(i) Find the initial displacement of the particle from O. [1]

(ii) Find the values of t when the particle is instantaneously at rest. [4]

(iii) Find the value of t when the acceleration of the particle is zero. [5]

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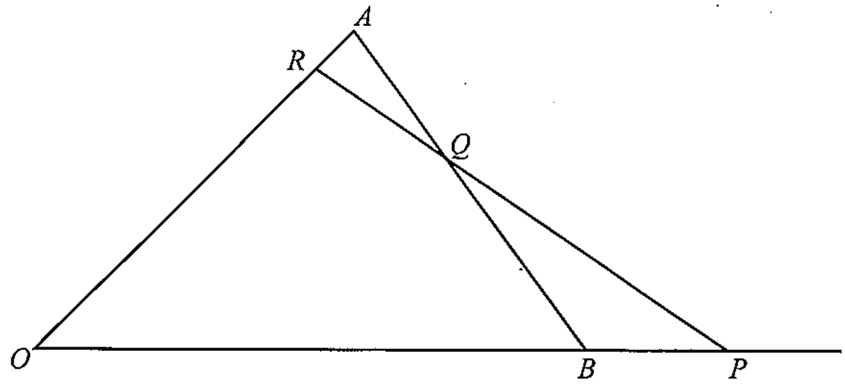
1



The position vectors of points A and B relative to an origin O are **a** and **b** respectively. The point P is such that $\overrightarrow{OP} = \frac{5}{4}\overrightarrow{OB}$. The point Q is such that $\overrightarrow{AQ} = \frac{1}{3}\overrightarrow{AB}$. The point R lies on OA such that RQP is a straight line where $\overrightarrow{OR} = \lambda\overrightarrow{OA}$ and $\overrightarrow{QR} = \mu\overrightarrow{PR}$.

(i) Express $\overrightarrow{OQ}$ and $\overrightarrow{PQ}$ in terms of **a** and **b**. [2]

(ii) Express $\overrightarrow{QR}$ in terms of λ, **a** and **b**. [2]

(iii) Express $\overrightarrow{QR}$ in terms of μ, **a** and **b**. [3]

(iv) Hence find the value of λ and of μ. [3]

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2 It is given that $\mathbf{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 21 \\ 2 \end{pmatrix}$.

(i) Find $|\mathbf{a} + \mathbf{b} + \mathbf{c}|$. [2]

(ii) Find λ and μ such that $\lambda \mathbf{a} + \mu \mathbf{b} = \mathbf{c}$. [3]

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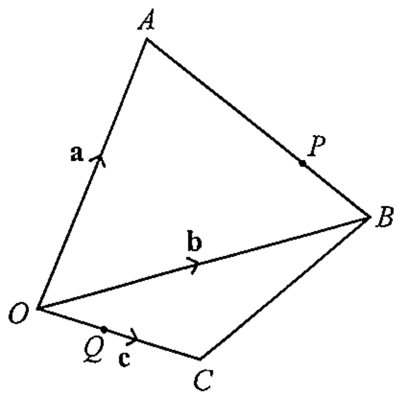
3



The figure shows points A, B and C with position vectors **a**, **b** and **c** respectively, relative to an origin O. The point P lies on AB such that $AP:AB = 3:4$. The point Q lies on OC such that $OQ:QC = 2:3$.

(i) Express $\overrightarrow{AP}$ in terms of **a** and **b** and hence show that $\overrightarrow{OP} = \frac{1}{4}(\mathbf{a} + 3\mathbf{b})$. [3]

(ii) Find $\overrightarrow{PQ}$ in terms of **a**, **b** and **c**. [3]

(iii) Given that $5\overrightarrow{PQ} = 6\overrightarrow{BC}$, find **c** in terms of **a** and **b**. [2]

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4 Vectors **a**, **b** and **c** are such that $\mathbf{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$.

(i) Show that $|\mathbf{a}| = |\mathbf{b} + \mathbf{c}|$. [2]

(ii) Given that $\lambda\mathbf{a} + \mu\mathbf{b} = 7\mathbf{c}$, find the value of λ and of μ. [3]

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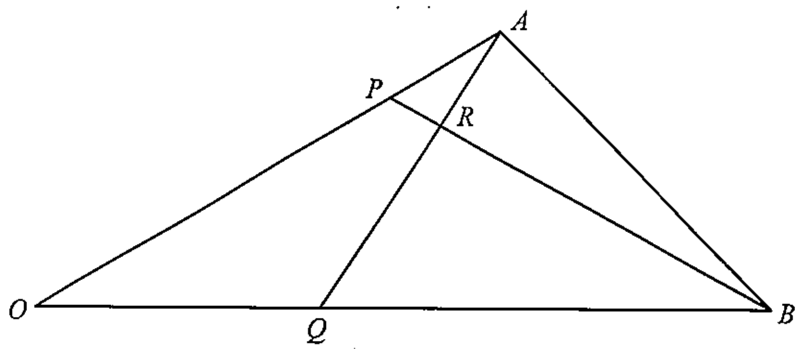
5



The position vectors of points A and B relative to an origin O are $\mathbf{a}$ and $\mathbf{b}$ respectively. The point P is such that $\overrightarrow{OP} = \mu\overrightarrow{OA}$. The point Q is such that $\overrightarrow{OQ} = \lambda\overrightarrow{OB}$. The lines AQ and BP intersect at the point R.

(i) Express $\overrightarrow{AQ}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) Express $\overrightarrow{BP}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [1]

It is given that $3\overrightarrow{AR} = \overrightarrow{AQ}$ and $8\overrightarrow{BR} = 7\overrightarrow{BP}$.

(iii) Express $\overrightarrow{OR}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [2]

(iv) Express $\overrightarrow{OR}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [2]

(v) Hence find the value of μ and of λ. [3]

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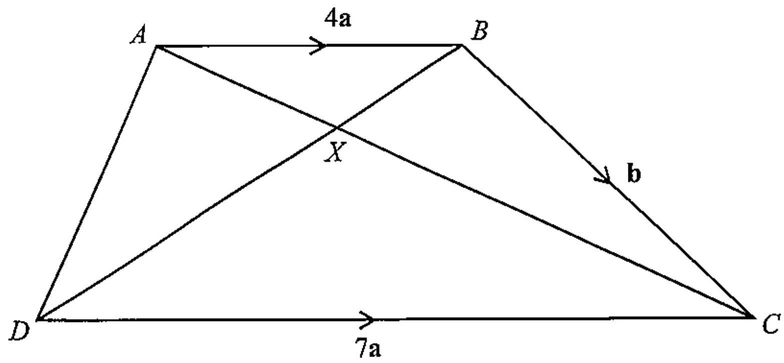
6



In the diagram $\overrightarrow{AB} = 4\mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{DC} = 7\mathbf{a}$. The lines AC and DB intersect at the point X. Find, in terms of **a** and **b**,

(i) $\overrightarrow{DA}$, [1]

(ii) $\overrightarrow{DB}$. [1]

Given that $\overrightarrow{AX} = \lambda\overrightarrow{AC}$, find, in terms of **a**, **b** and λ,

(iii) $\overrightarrow{AX}$, [1]

(iv) $\overrightarrow{DX}$. [2]

Given that $\overrightarrow{DX} = \mu\overrightarrow{DB}$,

(v) find the value of λ and of μ. [4]

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1

At 12 00 hours, a ship has position vector $(54\mathbf{i} + 16\mathbf{j})$ km relative to a lighthouse, where $\mathbf{i}$ is a unit vector due East and $\mathbf{j}$ is a unit vector due North. The ship is travelling with a speed of $20\,\text{km h}^{-1}$ in the direction $3\mathbf{i} + 4\mathbf{j}$.

(i) Show that the position vector of the ship at 15 00 hours is $(90\mathbf{i} + 64\mathbf{j})$ km. [2]

(ii) Find the position vector of the ship t hours after 12 00 hours. [2]

A speedboat leaves the lighthouse at 14 00 hours and travels in a straight line to intercept the ship. Given that the speedboat intercepts the ship at 16 00 hours, find

(iii) the speed of the speedboat, [3]

(iv) the velocity of the speedboat relative to the ship, [1]

(v) the angle the direction of the speedboat makes with North. [2]

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2 A pilot flies his plane directly from a point A to a point B, a distance of 450 km. The bearing of B from A is 030°. A wind of $80\,\text{km}\,\text{h}^{-1}$ is blowing from the east. Given that the plane can travel at $320\,\text{km}\,\text{h}^{-1}$ in still air, find

(i) the bearing on which the plane must be steered, [4]

(ii) the time taken to fly from A to B. [4]

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3 In this question **i** is a unit vector due East and **j** is a unit vector due North.
At 12 00 hours, a ship leaves a port P and travels with a speed of $26\,\text{kmh}^{-1}$ in the direction $5\mathbf{i} + 12\mathbf{j}$.

(i) Show that the velocity of the ship is $(10\mathbf{i} + 24\mathbf{j})\,\text{kmh}^{-1}$. [2]

(ii) Write down the position vector of the ship, relative to P, at 16 00 hours. [1]

(iii) Find the position vector of the ship, relative to P, t hours after 16 00 hours. [2]

At 16 00 hours, a speedboat leaves a lighthouse which has position vector $(120\mathbf{i} + 81\mathbf{j})$ km, relative to P, to intercept the ship. The speedboat has a velocity of $(-22\mathbf{i} + 30\mathbf{j})\,\text{kmh}^{-1}$.

(iv) Find the position vector, relative to P, of the speedboat t hours after 16 00 hours. [1]

(v) Find the time at which the speedboat intercepts the ship and the position vector, relative to P, of the point of interception. [4]

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